19/10

SOH CAH TOA

Name: Claire Li

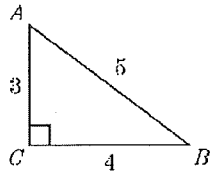
Date: April 1st

Math 9 Enriched: Section 6.1 Basic Trigonometry Ratios:

1. Find the ratios of the following functions and then solve for the angle:

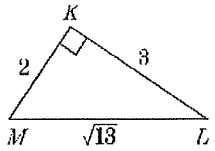
a) $\sin A =$ Opp/Hyp

$= \frac{4}{5}$



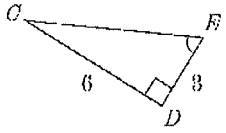
b) $\cos M =$ Adj/Hyp

$= \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$



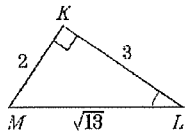
c) $\tan E =$ Opp/Adj

$= \frac{6}{3} = 2$



d) $\sin L =$ Opp/Hyp

$= \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$



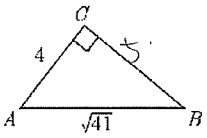
e) $\tan A =$ Opp/Adj

$= \frac{5}{4}$

$BC^2 = AB^2 - AC^2$

$BC^2 = 41 - 16$

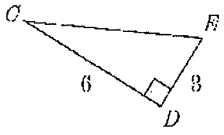
$BC = 5$



f) $\cos C =$ Adj/Hyp

$= \frac{6}{3\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$CE = \sqrt{6^2+3^2} = 3\sqrt{5}$

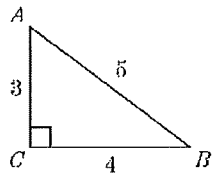


2. Indicate which angle is equal to "*x*" and then solve for the angle:

a) $\sin x = 0.6$

$\sin x = \frac{3}{5}$

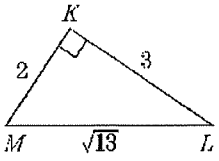
$x = B$



b) $\sin x = \frac{3\sqrt{13}}{13}$

$\sin x = \frac{3}{\sqrt{13}}$

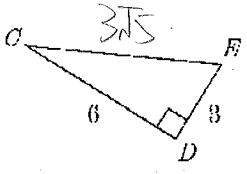
$x = M$



c) $\cos x = \frac{\sqrt{5}}{5}$

$\cos x = \frac{3}{3\sqrt{5}}$

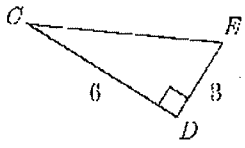
$x = E$



d) $\tan x = 2$

$\tan x = \frac{6}{3}$

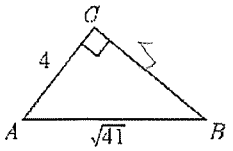
$x = E$



e) $\tan x = 0.8$

$\tan x = \frac{4}{5}$

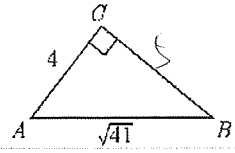
$x = B$



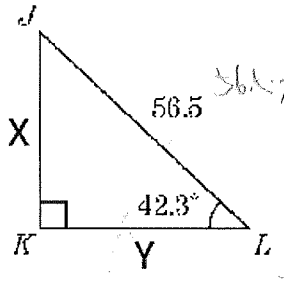
f) $\sin x = \frac{5\sqrt{41}}{41}$

$\sin x = \frac{5}{\sqrt{41}}$

$x = A$



3. Find the length of the missing sides:



$\sin L = \frac{x}{56.5}$

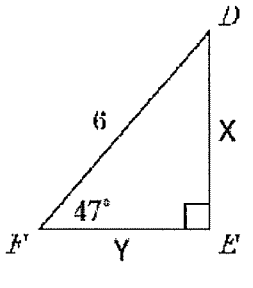
$56.5 \times \sin 42.3^\circ = x$

$x = 38.03$

$\cos L = \frac{y}{56.5}$

$56.5 \times \cos 42.3^\circ = y$

$41.79 = y$



$\sin F = \frac{x}{6}$

$6 \cdot \sin 47^\circ = x$

$x = 4.39$

$\cos F = \frac{y}{6}$

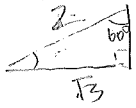
$6 \cdot \cos 47^\circ = y$

$y = 6.09$

4. When I use the regular trigonometric functions like sine, cosine, and tangent, does it only work for right triangles? Or can I use it for all sorts of triangles?

Only right triangles.

5. When I sine an angle like 60°, it gives me a value like 0.866025403. What does this number represent?



It represents the ratio of the opp side to the hypotenuse which is $\frac{\sqrt{3}}{2}$

6. When I cosine or sine any angle in a right triangle will I ever get a value greater than 1? Why or why not?

$\cos\theta = \frac{adj}{hyp}$ $\sin\theta = \frac{opp}{hyp}$ In a Rt$\triangle$, hyp > adj or opp sides.

∴ No

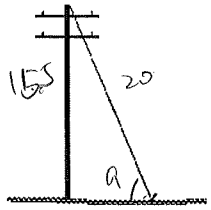
7. What does the inverse trigonometric function do? Ie: $\sin^{-1}, \cos^{-1}$, or $\tan^{-1}$. What is the purpose of these inverse functions? To find angles when we know the ratio.

eg. $\sin^{-1}(0.5) = 30^\circ$

8. When I take sine 45 and divide it by cosine 45, does it equal to tangent 45? Why is it equal? Does sine an angle divided by cosine an angle always to tangent the angle? Why or why not?

$\sin 45^\circ = \frac{Opp}{Hyp}$ $\cos 45^\circ = \frac{Adj}{Hyp}$ $\frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\frac{Opp}{Hyp}}{\frac{Adj}{Hyp}} = \frac{Opp}{Adj} = \tan 45^\circ$

9. A 20meter long wire is attached to the top of a telephone pole 15.5 meters tall. What is the exact measure of the angle the wire makes with the ground?

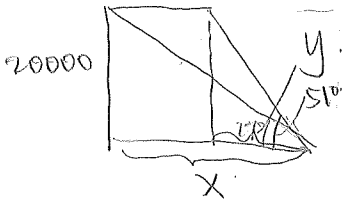


$\sin \angle a = \frac{15.5}{20}$

$\sin^{-1}\left(\frac{31}{40}\right) = a$

$a = 51^\circ$

10. An airplane is flying at a constant altitude of 20,000 meters directly towards an observer stationed on the ground. AT 2:00pm, the observed angle of elevation of the plane is 27°. One minute later, the angle of elevation is 51°. What is the plane's approximate ground speed in miles per hour?



$\tan 27^\circ = \frac{20000}{x}$

$x = 39252.21$

$\tan 51^\circ = \frac{20000}{y}$

$y = 16195.68$

$x - y = 23056.53$

$\frac{23056.53 \times 60}{1609.34} = 856.90 \text{ m/h}$

11. At a point 30m from a building, the angle of elevation to the top of the building is $70°$. If the observer's eyes is 1.2m above ground, how tall is the building?

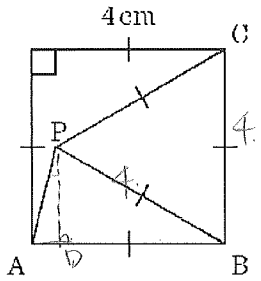
$\tan 70° = \frac{h}{30}$

$h = \tan 70° \cdot 30$

$h = 82.42$

∴ The building is 83.62 m tall

12. ABCD is a square with side length 4cm and ΔPBC is equilateral as shown. Determine the length of AP to the nearest tenth of a centimeter.



$\angle APB = \frac{180-30}{2} = 75°$

$\angle APD = 75° - \angle DPB = 75° - 60° = 15°$

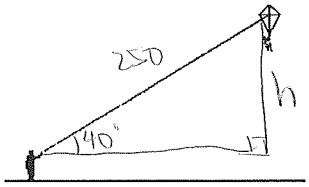
$\cos 15° = \frac{DP}{AP}$

$AP \cdot \cos 15° = DP$

$AP \cdot 0.965 = 2$

$AP = 2.1$

13. When I went kite flying the other day, I managed to let out an entire roll of string (250meters). If the string, when pulled tight, formed a $40°$ angle with the ground, about how high was the kite?



$\sin 40° = \frac{h}{250}$

$160.70 = h$

14. Calculate the perimeter of a regular pentagon inscribed in a circle with a radius of 6.5cm.

$\frac{360}{5} = 72°$

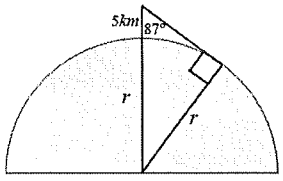
$\frac{180-72}{2} = 54°$

$\frac{\sin 54°}{6.5} = \frac{\sin 72°}{x}$

$x = 7.64$

$7.64 \times 5 = 38.21$ cm

15. Calculating the Earth's circumference involves the use of trigonometry. Imagine that an observer is standing on the top of a 5km tall mountain and is looking into the horizon at an angle of $88°$. The true vertical is an imaginary line that connects to the top of the mountain to the center of the earth. Use the diagram to calculate the radius and circumference of the earth.



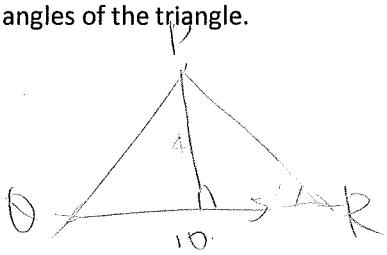
$\sin 87° = \frac{r}{r+5}$

$\sin 87° r + \sin 87° 5 = r$

$5 \sin 87° = (1 - \sin 87°) r$

$r = 3643.40$

16. An isosceles ΔPQR has a base QR, of length 10cm and a height of 4cm. Find the measures of the three angles of the triangle.



$\tan \angle PRQ = \frac{4}{5}$

$\angle PRQ = 39°$

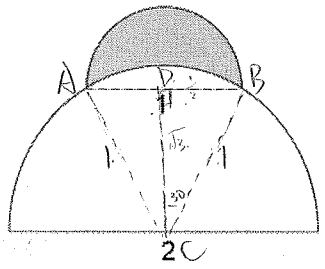
$\angle QPR = 180 - 2\angle PRQ = 102°$

$\angle QPR = 102°$

$\angle PRQ = 39°$

$\angle PQR = 39°$

17. A semicircle of diameter 1 sits at the top of a semicircle of diameter 2 as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a lune. Determine the area of this lune.



$S_{semicircle} = \frac{1}{2}\cdot\pi\cdot\left(\frac{1}{2}\right)^2 = \frac{\pi}{8}$

$S_{\triangle ABC} = \frac{\sqrt{3}}{4}$

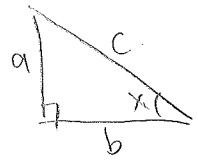
$\frac{\pi}{8} + \frac{\sqrt{3}}{4} - \frac{\pi}{6} = \frac{\sqrt{3}}{4} + \frac{3\pi - 4\pi}{24}$

(A) $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{4} - \frac{1}{12}\pi$ (C) $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ (D) $\frac{\sqrt{3}}{4} + \frac{1}{24}\pi$ $= \frac{\sqrt{3}}{4} - \frac{\pi}{24}$

(E) $\frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

$\frac{60}{360}\cdot\pi\cdot 1 = \frac{\pi}{6}$

18. When given a right triangle with one of the angles "X", what is the value of $(\sin x)^2 + (\cos x)^2$? Does this value change for different values of the angle? Explain your reasoning.



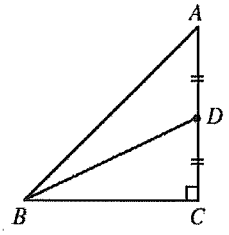
$(\sin x)^2 + (\cos x)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$

$= \frac{a^2+b^2}{c^2}$

$= \frac{c^2}{c^2} = 1$

Doesn't change.

19. In the diagram, $AD = DC$, $\sin\angle DBC = 0.6$ and $\angle ACB = 90°$. What is the value of $\tan\angle ABC$?



$\sin\angle DBC = 0.6$

$\frac{DC}{BD} = 0.6$

$\frac{DC}{BD} = \frac{3}{5}$

Let $DC = 3x$, $BD = 5x$.

$\therefore BC = 4x$

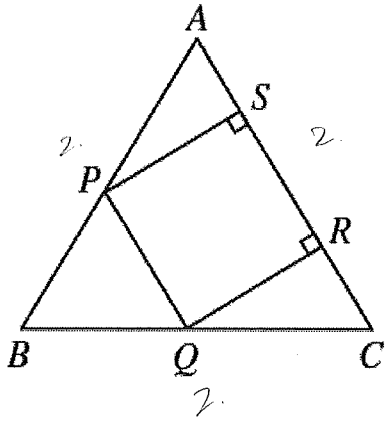
$\tan\angle ABC = \frac{AC}{BC}$

$= \frac{3x \times 2}{4x}$

$= \frac{6x}{4x}$

$= \frac{3}{2}$

20. Challenge: An equilateral triangle ABC has side lengths 2. A square, PQRS, is such that P lies on AB, Q lies on BC, and R and S lie on AC as shown. The points P, Q, R, and S move so that P, Q, and R always remain on the sides of the triangle and S moves from AC to AB through the interior of the triangle. If the points P, Q, R and S always form the vertices of a square, show that the path traced out by S is a straight line parallel to BC. Most students will find this question too challenging. (Hint: the size of the square changes.)



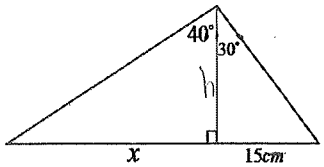
Name: Claire Date: April 9

Math 9 Enriched: Section 6.2 Trigonometric Problems

10.163
16.8827

1. Calculate the length of the missing side "x" in each of the following triangles:

a)



$\tan 30° = \frac{15}{h}$

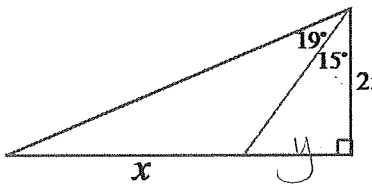
$h = \frac{15}{\tan 30°}$

$\tan 40° = \frac{x}{h}$

$x = h \cdot \tan 40°$

$x = 21.80$

b)



$\tan 15° = \frac{y}{25}$

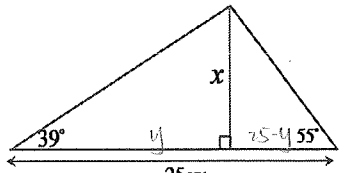
$\tan 15° \times 25 = y$

$\tan 34° = \frac{x+y}{25}$

$25 \tan 34° = x + y$

$x = 10.16$

C)



$\frac{x}{\tan 39° + \tan 55°} = 25$

$\frac{(\tan 39° + \tan 55°)x}{\tan 39° \tan 55°} = 25$

$x = 12.92$

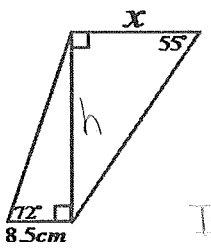
$\tan 39° = \frac{x}{y}$

$\frac{x}{\tan 39°} = y$

$\tan 55° = \frac{x}{25-y}$

$\frac{x}{\tan 55°} = 25 - y$

d)



$\tan 72° = \frac{h}{8.5}$

$h = \tan 72° \cdot 8.5$

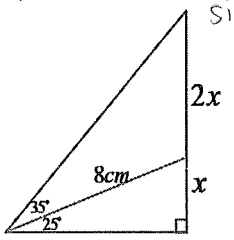
$\tan 55° = \frac{h}{x}$

$\frac{h}{\tan 55°} = x$

$\frac{\tan 72° \cdot 8.5}{\tan 55°} = x$

$x = 22.21$

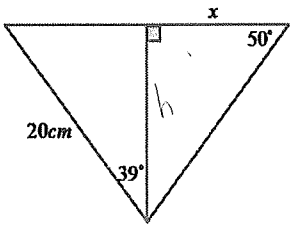
e)



$\sin 25° = \frac{x}{8}$

$x = 3.38$

f)



$\cos 39° = \frac{h}{20}$

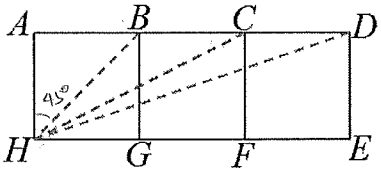
$h = 20 \cdot \cos 39°$

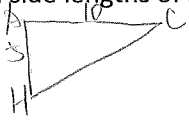
$\tan 50° = \frac{h}{x}$

$\frac{h}{\tan 50°} = x$

$x = \frac{\cos 39° \cdot 20}{\tan 50°} = 13.04$

2. In the diagram, three squares with side lengths of 5cm are connected together. What is the degree value of $\angle BHC$ and $\angle DHC$?





$\tan \angle AHC = \frac{10}{5}$

$\tan^{-1}(2) = \angle AHC$

$\angle AHC = 63.4349$

$\therefore \angle BHC = \angle AHC - \angle AHB = 18.43$ $\angle BHC$

$\tan \angle AHD = \frac{15}{5}$

$\tan^{-1}(3) = \angle AHD$

$\angle AHD = 71.565°$

$\therefore \angle AHD - \angle AHC = \angle DHC = 8.13$ $\angle DHC$

3. In ΔABC, $AB = AC$, $\angle C = 70°$, and $BC = 4$cm. Calculate the height of the triangle to the nearest mm.

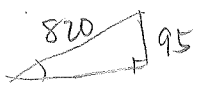
$\tan \angle C = \frac{h}{2}$

$2 \cdot \tan 70° = h$

$h = 5.49$

4. A wheelchair ramp is 8.2m long. It rises 95cm. Determine the angle of inclination of the ramp to 1 decimal place.

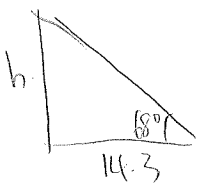
8.2m = 820cm



$\sin \angle a = \frac{95}{820}$

$\angle a = 6.7°$

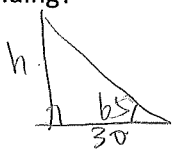
5. The angle of elevation of the sun is $68°$ when a tree casts a shadow 14.3m long. How tall is the tree?



$\tan 68° = \frac{h}{14.3}$

$h = 35.39$ m

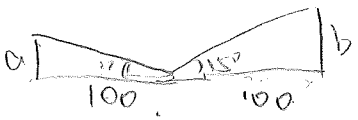
6. At a point 30m from a building, the angle of elevation to the top of the building is 65°. The observer's eyes are 1.5m above the ground. How tall is the building? How far are the observer's eyes from the top of the building?



$\tan 65^\circ = \frac{h}{30}$

$h = 64.34$

7. Two trees are 200m apart. A man standing in the middle between them is looking at their tops and notice that the angle of elevations are 15° and 12° respectively. How much taller is the larger tree?



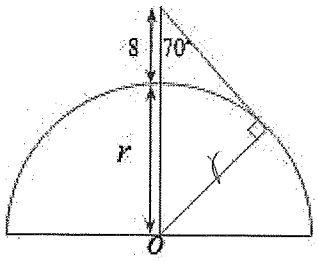
Let the height of the shorter tree be a,
taller tree be b

$\tan 12^\circ = \frac{a}{100}$ $\tan 15^\circ = \frac{b}{100}$

$a = 21.26$ $b = 26.79$

$b - a = \boxed{5.54}$

8. Given the following semi-circle, find the length of the radius:



$\sin 70^\circ = \frac{r}{8+r}$

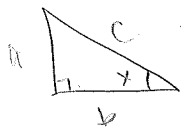
$\sin 70^\circ (8+r) = r$

$8 \cdot \sin 70^\circ = (1 - \sin 70^\circ) r$

$r = \frac{8 \cdot \sin 70^\circ}{1 - \sin 70^\circ}$

$r = \boxed{24.65}$

9. What is $\sin x(\tan x) + \frac{\cos x}{\tan x}$ equal to?

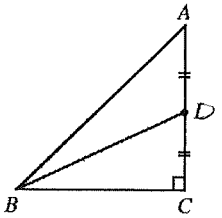


$= \frac{a}{c} \cdot \frac{a}{b} + \frac{\frac{b}{c}}{\frac{a}{b}}$

$= \frac{a^2}{bc} + \frac{b^2}{ac}$

$= \frac{a^3 + b^3}{abc}$

10. In the diagram, AD=DC, $\sin \angle DBC = 0.6$ and $\angle ABC = 90^\circ$. What is the value of $\tan \angle ABC$?

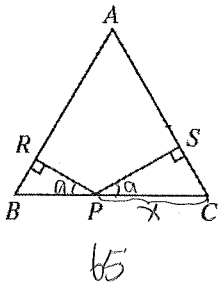


$\sin \angle DBC = \frac{DC}{DB}$

$0.6 = \frac{DC}{DB}$

$\frac{3}{5} = \frac{DC}{DB}$

11. Triangle ABC is isosceles with AB=AC and BC=65cm. P is a point on BC such that the perpendicular distances from P to AB and AC are 24cm and 36cm, respectively. Find the area of ΔABC in cm^2.



Let $\angle SPC = \angle RPB = \angle a$

$PC = x$, $BP = 65 - x$

$\cos a = \frac{36}{x}$ $\cos a = \frac{24}{65-x}$

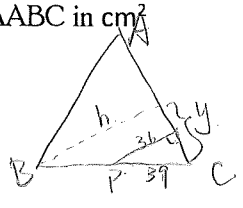
$\therefore \frac{36}{x} = \frac{24}{65-x}$

$2340 - 36x = 24x$

$2340 = 60x$

$x = 39$

$\therefore SC = 15$



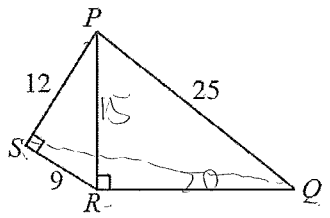
$\frac{h}{65} = \frac{36}{39}$

$h = 60$

$\sqrt{65^2 - 60^2} = 25$

$25 \times 60 = \boxed{1500}\ cm^2$

12. In The diagram, ΔPSR is right-angled at S and ΔPRQ is right angled at R. Also, PS =12, SR=9, and PQ = 25.
a) Determine the length of RQ, b) Determine the area of figure PQRS, c) Show that $\angle QPR = \angle PRS$
d) Determine the length of SQ



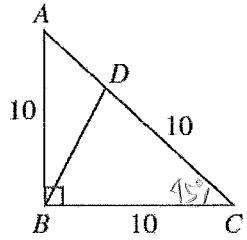
a) RQ = 20.

b) $A_{PQRS} = A_{\triangle PSR} + A_{\triangle PRQ}$
$= \frac{12\times 9}{2} + \frac{15\times 20}{2}$
$= 54 + 150 = 204$

c). $\because \frac{PS}{SP} = \frac{RQ}{RP}$ & $\frac{PR}{SR} = \frac{PQ}{RP}$
$\therefore \triangle PSR \sim \triangle QRP$
$\therefore \angle PRS = \angle QPR$

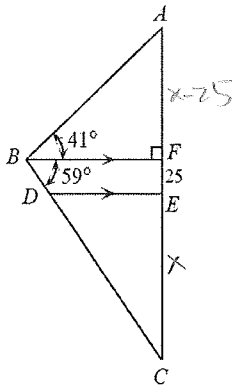
d. $\angle SPQ = 90°$
$SQ = \sqrt{PS^2 + PQ^2} = \sqrt{769}$

13. In the diagram, $\angle ABC = 90°$ and *AB = BC = CD = 10*. What is the length of AD?



$AC = \sqrt{2}AB = 10\sqrt{2}$
$AD = AC - DC = 10\sqrt{2} - 10 \approx \boxed{4.14}$

14. In the diagram, $\angle ABF = 41°$, $\angle FBC = 59°$, DE is parallel to BF, and EF =25. If AE=EC, determine the length of AE, to 2 decimal places.



Let AE = EC = x, AF = x − 25

$\tan 41° = \frac{x-25}{BF}$ $\quad \tan 59° = \frac{25+x}{BF}$

$BF = \frac{x-25}{\tan 41°}$ $\quad BF = \frac{25+x}{\tan 59°}$

$\therefore \frac{x-25}{\tan 41°} = \frac{x+25}{\tan 59°}$

$\tan 41°\,x + \tan 41°\cdot 25 = \tan 59°\,x - 25\cdot\tan 59°$

$(\tan 59° - \tan 41°)x = 25(\tan 41° + \tan 59°)$

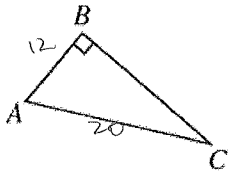
$x = \boxed{79.67}$

15. If $0° < x < 90°$ and $3\sin(x) - \cos(15°) = 0$, what is the value of "x" to the nearest tenth of a degree?

$3\sin(x) = \cos(15°)$
$\sin(x) = \frac{\cos(15°)}{3}$
$\sin^{-1}\left(\frac{\cos 15°}{3}\right) = x$

$x = \boxed{6.2}$

16. In the diagram, ΔABC is right-angled at B and AC=20. If $\sin C = \frac{3}{5}$, what is the length of BC?



$\sin C = \frac{AB}{AC}$

$\frac{AB}{AC} = \frac{3}{5}$

$\frac{AB}{20} = \frac{3}{5}$

$\therefore AB = 12$

$BC = \sqrt{20^2 - 12^2} = \boxed{16}$

17. Given that $3(\sin x) - 2 = 0$, solve for all the possible values of "x" for $0 < x < 180°$

$\sin x = \frac{2}{3}$

$x = \boxed{41.81}°$

18. Given that $6\sin^2 x + \sin x - 1 = 0$, solve for all the possible values of "x" for $0 < x < 180°$

let $\sin X = a$.

$6a^2 + a - 1 = 0$

$(3a-1)(2a+1) = 0$

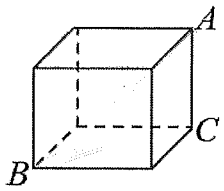
$3a \times -1$

$2a \times 1$

$\sin X = \frac{1}{3}$ or $\sin X = -\frac{1}{2}$

$X = 19.47$ $\quad X = -30$

19. Given the following cube with side length "x", two lines are drawn to connect Point "A" to "B" and then to "C". Calculate the value of $\angle ABC$ in terms of "x".



$BC = \sqrt{2}x$.

$AC = x$.

$\tan \angle ABC = \frac{AC}{BC}$

$\tan \angle ABC = \frac{1}{\sqrt{2}}$

$\angle ABC = 35.26$

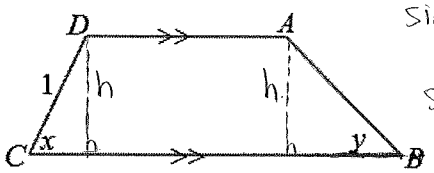
20. In the figure below, which of the following expressions is equal to side AB?

a) $\sin x \sin y$

b) $\frac{\sin x}{\sin y}$

c) $\frac{\sin y}{\sin x}$

d) $\frac{\cos y}{\cos x}$



$\sin y = \frac{h}{AB}$ $\quad AB = \frac{h}{\sin y}$

$\sin x = \frac{h}{1}$ $\quad AB = \frac{\sin x}{\sin y}$

21. Which of the following expressions below will give the area of parallelogram ABCD?

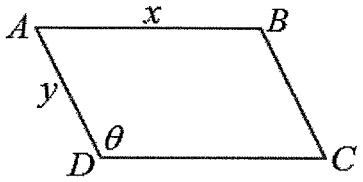
a) $\frac{1}{2}xy\sin\theta$

b) $xy\sin\theta$

c) $\frac{1}{2}xy\cos\theta$

d) $xy\cos\theta$

Area $= xy\sin\theta$



22. In the figure, if AX bisects $\angle BAC$, which expression is equal to $\frac{BX}{CX}$?

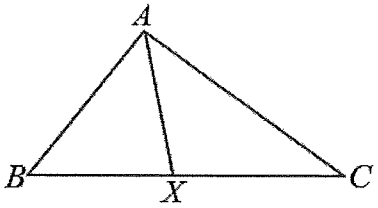
a) $\frac{\sin B}{\sin C}$

b) $\frac{\sin C}{\sin B}$

c) $\frac{\cos B}{\cos C}$

d) $\sin B \sin C$

$\frac{BX}{CX} = \frac{\sin B}{\sin C}$.



23. If $A + B = 180°$, then which of the following statements will be true?

i) $\sin A = \sin B$ — true

ii) $\cos A = \cos B$ — false

iii) $\tan A = -\tan B$ — true

iv) $\sin A = \cos B$ — false

24. In the figure below, AP and AQ are tangents to the circle with radius "r". Which expression is equal to AP?

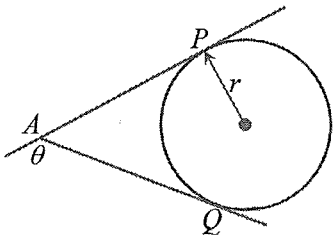
a) $r\tan\frac{\theta}{2}$

b) $\frac{r}{\tan\frac{\theta}{2}}$

c) $\frac{r}{\tan\theta}$

d) $r\sin\frac{\theta}{2}$

$AP = r\tan\frac{\theta}{2}$

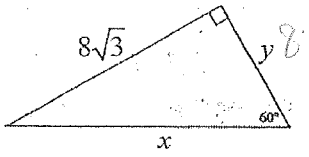


Name: Claire Li Date: April 15th

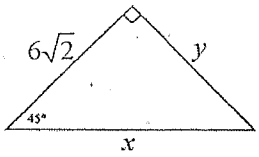
M9 Enriched: Section 6.3 Special Triangles: 30-60-90 & 45-45-90

1. Find the values of the missing sides "x" and "y"

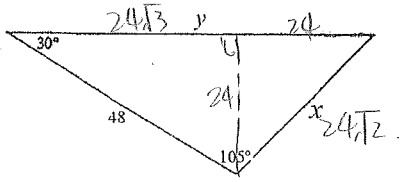
a) $x =$ 16 $y =$ 8



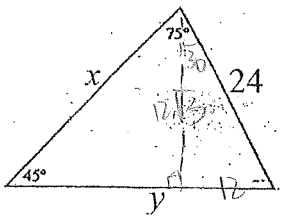
b) $x =$ 12 $y =$ $6\sqrt{2}$



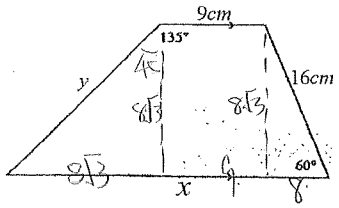
c) $x =$ $24\sqrt{2}$ $y =$ $24\sqrt{3}+24$



d) $x =$ $12\sqrt{6}$ $y =$ $12\sqrt{3}+12$

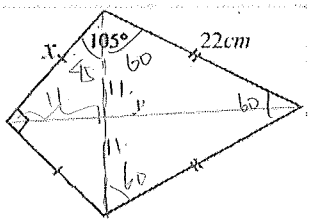


e) $x =$ $8\sqrt{3}+17$ $y =$ $8\sqrt{6}$



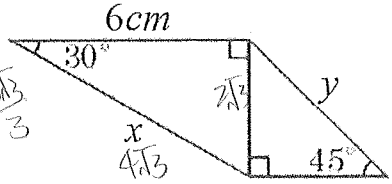
f) $x =$ $11\sqrt{2}$ $y =$ $11+11\sqrt{3}$

$x = \frac{22}{\sqrt{2}} = 11\sqrt{2}$

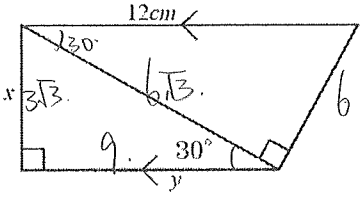


g) $x =$ $4\sqrt{3}$ $y =$ $2\sqrt{6}$

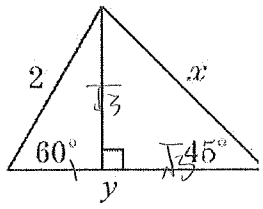
$\frac{6}{\sqrt{2}} = \frac{4\sqrt{3}}{3}$



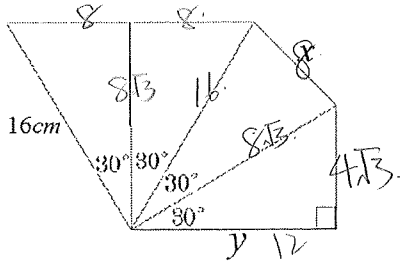
h) $x =$ $3\sqrt{3}$ $y =$ 9



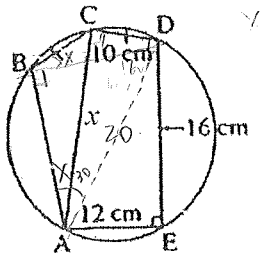
i) $x =$ $\sqrt{11}$ $y =$ $1+\sqrt{3}$



J) $x =$ 8 $y =$ 12

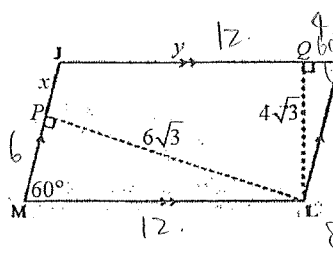


k) $x = 10\sqrt{3}$. $\angle BAC =$



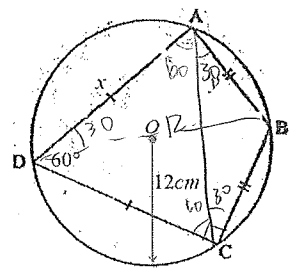
L) $x =$ 2 $y =$ 8

$y = 12 - 4 = 8$



$8 - 6 = 2$

m) $x =$ $6\sqrt{3}$ $y =$ 6



n) $x =$ 16.19

$x^2 + (x+11)^2 = 1001$

$x^2 + x^2 + 22x + 121 = 1001$

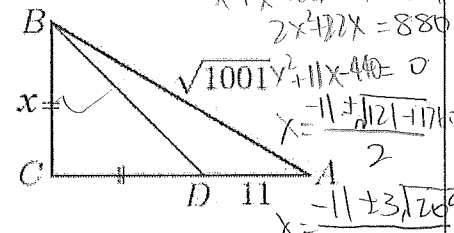
$2x^2 + 22x = 880$

$\sqrt{1001}y^2 + 11x - 440 = 0$

$x = \frac{-11 \pm \sqrt{121 + 17160}}{2}$

$x = \frac{-11 \pm 3\sqrt{2069}}{2}$.

$x = 16.19$.



o) $x =$ $2\sqrt{41}$

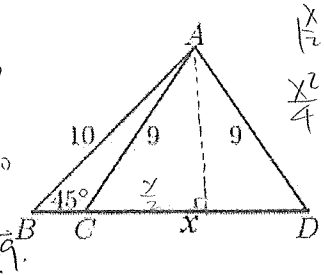
$(\frac{x}{2})^2 + (\frac{10}{\sqrt{2}})^2 = 9^2$

$\frac{x^2}{4} + 50 = 81$.

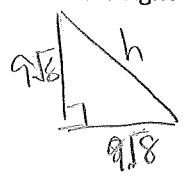
$\frac{x^2}{4} = 41$.

$x^2 = 4 \cdot 41$

$x = 2\sqrt{41}$.



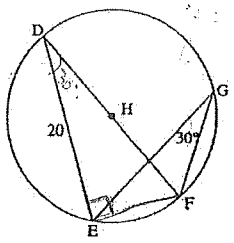
2. An isosceles right triangle has a leg of $9\sqrt{8}$. What is its perimeter?



$h = \sqrt{2} \cdot 9\sqrt{8} = 36$.

$P = 36 + 9\sqrt{8} \times 2 = \boxed{36 + 36\sqrt{2}}$

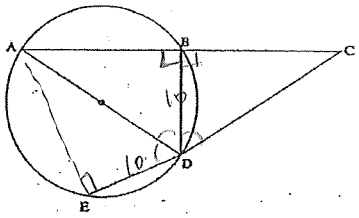
3. What is the length of DF?



$EF = \frac{DE}{\sqrt{3}} = \frac{20}{\sqrt{3}}$.

$DF = 2EF = \boxed{\frac{40}{\sqrt{3}}}$

4. If AB=BC, BD = ED = 10cm, find the length of AC.

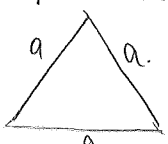


$\angle ADE = \angle ADB = \angle BDC = \frac{180°}{3} = 60°$.

$BC = 10 \times \sqrt{3} = 10\sqrt{3}$

$AC = 2BC = \boxed{20\sqrt{3}}$

5. The perimeter of an equilateral triangle is equal to its area. What is the length of the side of the triangle?



$P = 3a$.

$\text{Area} = \frac{\sqrt{3}a^2}{4}$

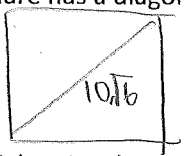
$3a = \frac{\sqrt{3}a^2}{4}$

$12 = \sqrt{3}a$.

$4\sqrt{3} = a$.

$\boxed{4\sqrt{3}}$

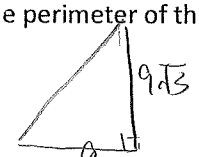
6. A square has a diagonal of length $10\sqrt{6}$. What is the perimeter of the square?



$\frac{10\sqrt{6}}{\sqrt{2}} = 10\sqrt{3}$.

$P = 10\sqrt{3} \times 4 = \boxed{40\sqrt{3}}$

7. In a right triangle, one leg is longer than the other leg by a factor of $\sqrt{3}$. If the longer leg is $9\sqrt{3}$ units long, what is the perimeter of the triangle?

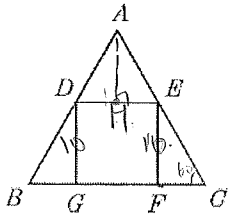


shorter leg $= \frac{9\sqrt{3}}{\sqrt{3}} = 9$.

$P = 18 + 9 + 9\sqrt{3} = \boxed{27 + 9\sqrt{3}}$

$\leftarrow 60° - 90° - 30°$ $\therefore h = 9 \times 2 = 18$.

8. $\triangle ABC$ is an equilateral triangle, and DEFG is a square of side 10cm. Find the length of a side of $\triangle ABC$.



$FC = \frac{EF}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$

$EC = 2FC = \frac{20\sqrt{3}}{3}$.

$HE = \frac{1}{2}DE = 5$.

$\therefore 60° - 90° - 30° \triangle$

$\therefore AE = 2HE = 10$.

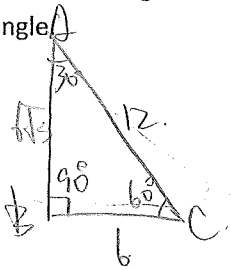
$AC = AE + EC = \boxed{10 + \frac{20\sqrt{3}}{3}}$

9. If the degree measures of the angles of a triangle are in the "x", "2x", "3x", and the longest side is 12cm long, then find the perimeter of the triangle.

$x + 2x + 3x = 180$

$6x = 180$

$x = 30$.

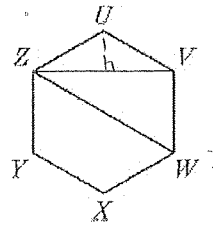


$BC = \frac{AC}{2} = 6$.

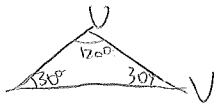
$AB = \sqrt{3} \cdot BC = 6\sqrt{3}$.

$\therefore P = 6\sqrt{3} + 6 + 12 = \boxed{18 + 6\sqrt{3}\text{ cm}}$

10. In the diagram, UVWXYZ is a regular hexagon with a perimeter of 42cm. what is the length of $\overline{ZV}$ and $\overline{ZW}$?

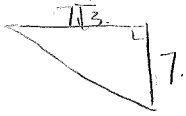


$\frac{42}{6} = 7$.



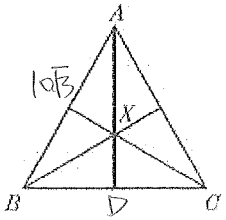
$ZV = \sqrt{3}UV = 7\sqrt{3}$

$ZW = 7 \times 2 = 14$



$\begin{cases} ZV = 7\sqrt{3} \\ ZW = 14 \end{cases}$

11. In the diagram, $\triangle ABC$ is an equilateral triangle with sides of length $10\sqrt{3}$. Point "x" is the centriod. What is the length of $\overline{AX}$?



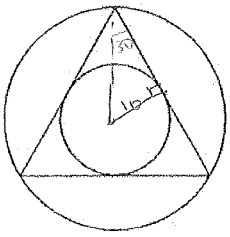
$\frac{AX}{XD} = \frac{2}{1}$

$BD = 5\sqrt{3}$.

$AD = 15$.

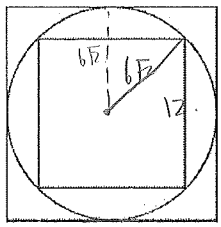
$\therefore AX = 15 \cdot \frac{2}{3} = \boxed{10}$

12. In the diagram, the smaller circle has a radius of 10cm. Determine the radius of the larger circle



$\therefore R = \boxed{20\text{cm}}$

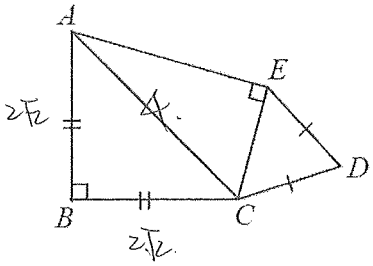
13. The small square has side length 12cm. Calculate the side length of the large square.



$r = \frac{12}{\sqrt{2}} = 6\sqrt{2}$.

$6\sqrt{2} \times 2 = \boxed{12\sqrt{2}}$

14. In the diagram, AB = BC = $AB = BC = 2\sqrt{2}$, $CD = DE$, $\angle CDE = 60°$ and $\angle EAB = 75°$. Determine the perimeter of figure ABCDE.



$\because CD = DE, \angle CDE = 60°$

$\therefore \triangle CDE$ is equilateral

$\angle CAE = \angle BAE - \angle BAC = 75° - 45° = 30°$

$AC = \sqrt{2}AB = 4$.

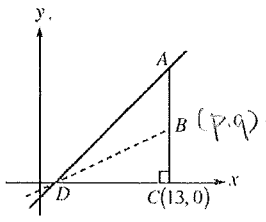
$EC = CD = DE = \frac{AC}{2} = 2$

$AE = \sqrt{3}CE = 2\sqrt{3}$.

$P_{ABCDE} = 2\sqrt{2} + 2\sqrt{2} + 2 + 2 + 2\sqrt{3}$

$= \boxed{4 + 4\sqrt{2} + 2\sqrt{3}}$

15. In the diagram, the equation of the line AD is $y=\sqrt{3}(x-1)$. BD bisects $\angle ADC$. If the coordinates of B are *(p,q)*, what is the exact value of *q*?



A(13, $12\sqrt{3}$)

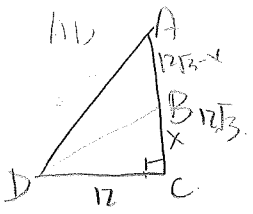
D(1 , 0)

$0=\sqrt{3}(x-1)$

$0=x-1$

$x=1$

$\therefore DC=12$



$\therefore AD=24$

$\frac{AB}{BC}=\frac{AD}{DC}$

$\frac{12\sqrt{3}-x}{x}=\frac{24}{12}$

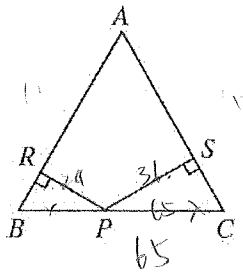
$2x=12\sqrt{3}-x$

$3x=12\sqrt{3}$

$x=4\sqrt{3}$

$\therefore B(13, 4\sqrt{3})$

16. Triangle ABC is isosceles with *AB = AC* and *BC = 65cm*. "P" is a point on *BC* such that the perpendicular distances from "P" to *AB* and *AC* are *24cm and 36cm* respectively. What is the area of triangle ABC?



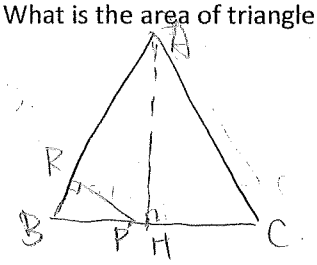
$\triangle BPR \sim \triangle CPS$

$\therefore \frac{RP}{BP}=\frac{SP}{PC}$

$\frac{24}{x}=\frac{36}{65-x}$

$36x=24\cdot 65-24x$

$60x=12\cdot 5\cdot 2\cdot 13$ $x=26$



$\triangle RBP \sim \triangle HBA$

$BH=\frac{65}{2}$

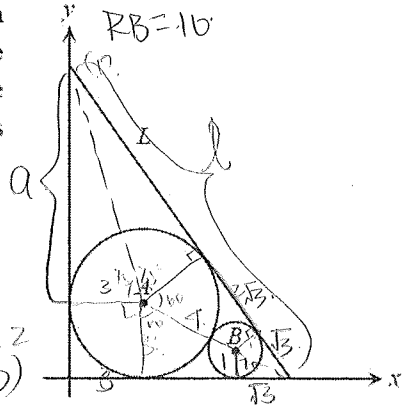
$AH=\frac{65}{2}\cdot\frac{12}{5}=78$

$S_{\triangle ABC}=\frac{65}{2}\cdot 78=2535$

15. The circle with centre *A* has radius 3 and is tangent to both the positive *x*-axis and positive *y*-axis, as shown. Also, the circle with centre *B* has radius 1 and is tangent to both the positive *x*-axis and the circle with centre *A*. The line *L* is tangent to both circles. The *y*-intercept of line *L* is

(A) $3+6\sqrt{3}$ (B) $10+3\sqrt{2}$ (C) $8\sqrt{3}$

(D) $10+2\sqrt{3}$ (E) $9+3\sqrt{3}$



$(a+3)^2+(3+2\sqrt{3}+\sqrt{3})^2=(a+3\sqrt{3})^2$

$a^2+6a+9+9+27+18\sqrt{3}=a^2+6\sqrt{3}a+27$

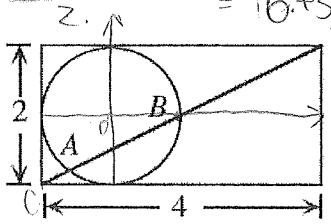
$3+3\sqrt{3}=(\sqrt{3}-1)a$

$a=\frac{3+3\sqrt{3}(\sqrt{3}+1)}{3-1}=\frac{3\sqrt{3}+3+9+3\sqrt{3}}{2}=6+3\sqrt{3}$

$6+3\sqrt{3}+3=9+3\sqrt{3}$

Challenge:

A circle is tangent to three sides of a rectangle having side lengths 2 and 4 as shown. A diagonal of the rectangle intersects the circle at points *A* and *B*. The length of *AB* is



(A) $\sqrt{5}$ (B) $\frac{4\sqrt{5}}{5}$ (C) $\sqrt{5}-\frac{1}{5}$

(D) $\sqrt{5}-\frac{1}{6}$ (E) $\frac{5\sqrt{5}}{6}$

B(1,0)

C(-1,-1)

$x^2+y^2=1$

$l_{AB}: y=\frac{1}{2}x-\frac{1}{2}$

Name: Claire Li Date: April 17

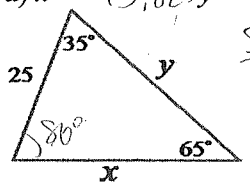
Math 9 Enriched: Section 6.4 Sine Law

1. Given each equation, solve for all values of θ where $0 \le \theta \le 180^\circ$

a) $\sin\theta = 0.25$ $\theta = 14^\circ$ or 166°	b) $\sin\theta = 0.85$ $\theta = 58^\circ$ or 122°	c) $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = 60^\circ$ or 120°	d) $\sin\theta = \frac{\sqrt{2}}{2}$ $\theta = 45^\circ$ or 135°
e) $\sin\theta = 1.2$ no solution.	f) $\sin\theta = -0.25$ $\theta = -14^\circ$ or -166°	g) $\sin\theta = 0$ $\theta = 0^\circ$	h) $\sin\theta = 1$ $\theta = 96^\circ$

2. Given each triangle, find the value of any missing side or angle "x" and "y"

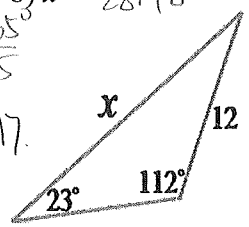
a) $x =$ 15.82 $y =$ 27.17



$\frac{\sin 65^\circ}{25} = \frac{\sin 35^\circ}{x}$ $\quad x = 19.82$

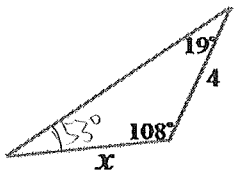
$\frac{\sin 80^\circ}{y} = \frac{\sin 65^\circ}{25}$ $\quad y = 27.17$

b) $x =$ 28.48



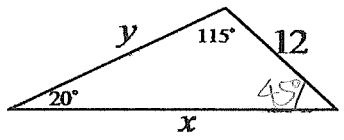
$\frac{\sin 112^\circ}{x} = \frac{\sin 23^\circ}{12}$ $\quad x = 28.48$

c) $x =$ 1.63



$\frac{\sin 153^\circ}{4} = \frac{\sin 19^\circ}{x}$ $\quad x = 1.63$

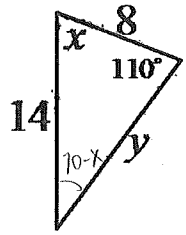
d) $x =$ 31.80 $y =$ 24.81



$\frac{\sin 115^\circ}{x} = \frac{\sin 20^\circ}{12}$ $\quad x = 31.80$

$\frac{\sin 45^\circ}{y} = \frac{\sin 20^\circ}{12}$ $\quad y = 24.81$

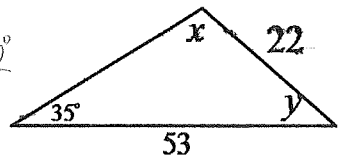
e) $x =$ 38 $y =$ 9.12



$\frac{\sin 110^\circ}{14} = \frac{\sin(70-x)^\circ}{8}$ $\quad 70 - x = 32$ $\quad x = 38$

$\frac{\sin 38^\circ}{y} = \frac{\sin 110^\circ}{14}$ $\quad y = 9.17$

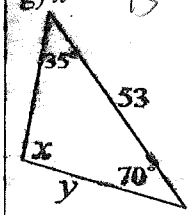
f) $x =$ $\quad y =$



$\frac{\sin x}{53} = \frac{\sin 35^\circ}{22}$ $\quad \sin x = 1.38$

no solution

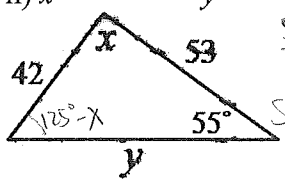
g) $x =$ 75° $y =$ 31.47



$x = 180^\circ - 35^\circ - 70^\circ = 75^\circ$

$\frac{\sin 75^\circ}{53} = \frac{\sin 35^\circ}{y}$ $\quad y = 31.47$

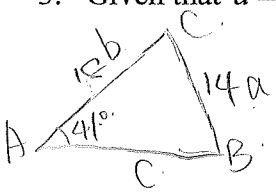
h) $x =$ $\quad y =$



$\frac{\sin 55^\circ}{42} = \frac{\sin(125-x)^\circ}{53}$ $\quad \sin(125-x) = 1.03$

no solution.

3. Given that $a = 14cm$, $b = 18cm$, and $\angle A = 41°$, find the area of $\triangle ABC$.

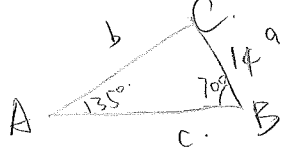


$\frac{\sin B}{18} = \frac{\sin 41°}{14}$ $\quad \frac{\sin 81°}{c} = \frac{\sin 41°}{14}$ $\quad s = \frac{a+b+c}{2} \approx 26.54$

$B = 58°$ $\quad c = 21.08$ $\quad \sqrt{s(s-a)(s-b)(s-c)}$

$\therefore C = 180° - 41° - 58° = 81°$ $\quad 116.03\ cm^2$

4. Given that ~~$a = 14cm$~~, $\angle B = 70°$, and $\angle A = 35°$, find the area of $\triangle ABC$.



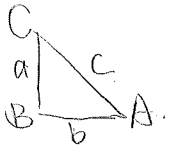
$\frac{\sin 76°}{b} = \frac{\sin 35°}{14}$ $\quad \frac{\sin 75°}{c} = \frac{\sin 35°}{14}$ $\quad s = \frac{a+b+c}{2} \approx 30.26$

$b = 29.94$ $\quad c = 23.58$ $\quad \sqrt{s(s-b)(s-b)(s-c)}$

$\therefore C = 180° - 35° - 70° = 75°$ $\quad = 155.06\ cm^2$

5. Which of the following expressions are true?

a. $\sin a = \sin(a + 180°)$ $\Rightarrow \sin a = -\sin(a + 180°)$

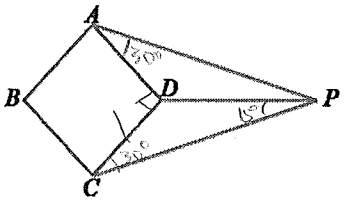


b. $\sin a = \sin(180° - a)$ ✓

c. $\sin(2a) = 2(\sin a)(\cos a)$ ✓

$= 2 \cdot \frac{a}{c} \cdot \frac{b}{c} = \frac{2ab}{c^2}$

6. In the diagram, ABCD is a square with sides of length 1 unit. Point P is such that $\angle PAD = \angle PCD = 30°$. Calculate the length of PD and PC to 1 decimal place

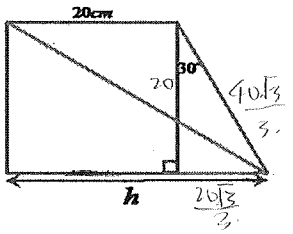


$\angle ADP = \angle CDP = \frac{360° - 90°}{2} = 135°$

$\frac{\sin 15°}{1} = \frac{\sin 30°}{PD}$ $\quad \frac{\sin 135°}{PC} = \frac{\sin 15°}{1}$

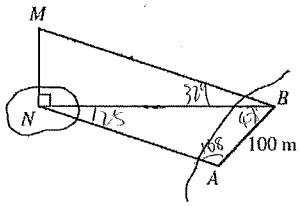
$PD = 1.9$ $\quad PC = 2.7$

7. Calculate the length of "x" to 1 decimal place



$20 + \frac{20\sqrt{3}}{3} = 31.5\ cm$

8. In determining the height, MN, of a tower ~~on an island~~, two points "A" and "B", 100m apart, are chosen on the same horizontal plane as "N". If $\angle NAB = 108°$, $\angle ABN = 47°$ and $\angle MBN = 32°$, determine the height of the tower to the nearest meter.



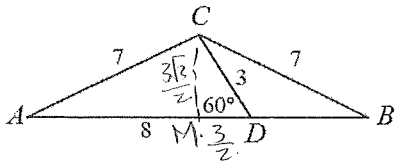
$\frac{\sin 25°}{100} = \frac{\sin 108°}{BN}$

$BN = 225.04m$

$\tan 32° = \frac{MN}{225.04}$

$MN = 141m$

9. In the diagram, $\triangle ABC$ is isosceles with AC=BC=7. Point D is on AB with $\angle CDA = 60^\circ$ $AD = 8$, and $CD = 3$. Determine the length of BD.



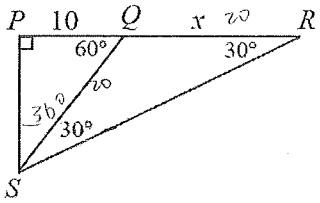
$MD = \frac{3}{2}$

$CM = \frac{3\sqrt{3}}{2}$

$MB = \sqrt{49 - \frac{27}{4}}$

$BD = MB - MD = \frac{8}{2} - \frac{3}{2} = \boxed{5}$ //

10. In the diagram, $PQ = 10$ and $QR = x$. Find the value of "x"

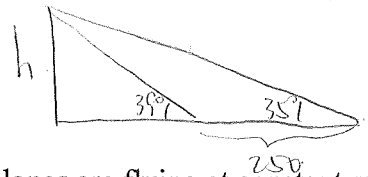


$SQ = 2PQ$ 20.

$SQ = QR = 20$

$\therefore x = \boxed{20}$ //

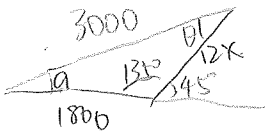
11. An observer is looking at a mountain peek at an angle of elevation of 35degrees. He walks 250meters towards the mountain and the angle of elevation to the peak is 39degrees. What is the height of the mountain?



$\frac{h}{\tan 35^\circ} - \frac{h}{\tan 39^\circ} = 250$

$h = \boxed{1294\text{m}}$ //

12. Two planes are flying at constant speeds in different directions from the same location. The first plane is flying due west at 150km/hr. The second plane is flying 45degrees East of North. How fast is the second plane flying if they are 3000km apart after 12 hours?



$150 \times 12 = 1800$ km.

$\frac{\sin 135^\circ}{3000} = \frac{\sin\theta}{1800}$

$\theta = 25.1^\circ$

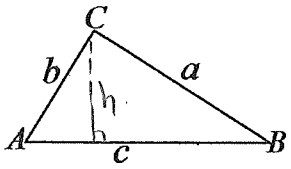
$\therefore \angle a = 180^\circ - 135^\circ - \theta$

$= 19.9^\circ$

$\frac{\sin 25.1^\circ}{1800} = \frac{\sin 19.9^\circ}{12x}$

$x = \boxed{120.36\ \text{km/h}}$ //

13. Without using the sine law, use the following triangle to prove that $b \times \sin A = a \times \sin B$. Show all your steps and reasoning.

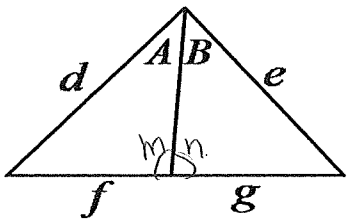


$b \times \sin A = b \times \frac{h}{b} = h.$

$a \times \sin B = a \times \frac{h}{a} = h.$

$\therefore b \times \sin A = a \times \sin B.$

14. Given the following triangle, prove that if $\angle A = \angle B$, then $f = \frac{g \times d}{e}$



$\frac{\sin A}{f} = \frac{\sin m}{d} \Rightarrow f = \frac{d \cdot \sin A}{\sin m}$ ①

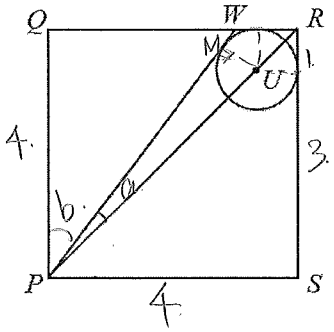
$\frac{\sin B}{g} = \frac{\sin n}{e} \Rightarrow g = \frac{e \cdot \sin B}{\sin n}$ ②

$\therefore m + n = 180°$ if $\angle A = \angle B$, $\sin A = \sin B$.

$\therefore \sin \angle m = \sin \angle n$

$\frac{①}{②} : \frac{f}{g} = \frac{d}{e} \Rightarrow f = \frac{g \times d}{e}$ $\therefore$ QED.

15. Square PQRS has side length 4m. Point U is on PR with $PR = 4UR$. A circle centered at U touches ~~two~~ sides of the square. PW is a tangent to the circle, with W on QR. What is the length of PW, to the nearest thousandth of a meter.



$PR = 4\sqrt{2}$

$UR = \sqrt{2}$

$MU = r = 1$

$PU = 3\sqrt{2}$

$\sin \angle a = \frac{MU}{PU}$

$\angle a = 13.63°$

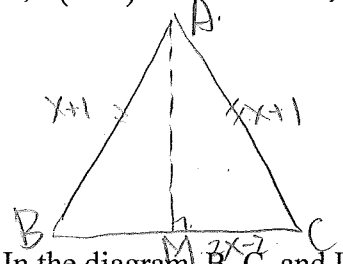
$\angle b = 45° - \angle a = 31.37°$

$\cos b = \frac{4}{PW}$

$PW = 4.68m$ //

16. In triangle ABC, $AB = AC = x + 1$ and $BC = 2x - 2$, where $x > 1$, then which of the following expressions is equal to the area of the triangle.

a) $(x-1)\sqrt{2x^2+2}$ **b)** $2(x-1)$ *c)* $\frac{1}{2}(x+1)^2$ *d)* $(x-1)(x+1)$ *e)* $2(x-1)\sqrt{x}$



$AM = \sqrt{(x+1)^2 - (x-1)^2}$

$= \sqrt{x^2+2x+1 - x^2+2x+1}$

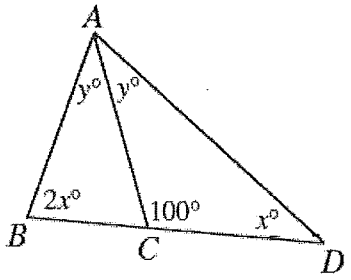
$= \sqrt{4x}$

$= 2\sqrt{x}$

Area $= \frac{2\sqrt{x} \cdot (2x-2)}{2}$

$= 2(x-1)\sqrt{x}$ //

17. In the diagram, B, C, and D lie on a straight line, with $\angle ACD = 100°$, with $\angle ADB = x°$, $\angle ABD = 2x°$, and $\angle DAC = \angle BAC = y°$. What is the value of x?



$2x + x + 2y = 180°$

$\begin{cases} 3x + 2y = 180° \\ x + y = 80° \end{cases}$

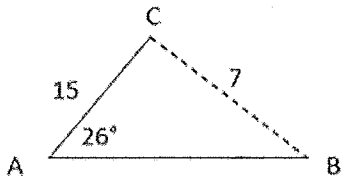
$x = 20°$ //

Name: Claire Li Date: April 23.

Math 9/10 Enriched: Section 6.4b Ambiguous Case with the Sine Law

1. Given each triangle, find the indicated angle to the nearest degree:

a) Find all the possible values for $\angle C$



ambiguous

$\frac{\sin B}{15} = \frac{\sin 26°}{7}$

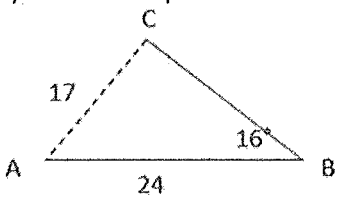
$\sin B° = 0.939$

$B \approx 70°$ or $110°$

$\angle C = 180° - 26 - 70° = 84°$

or $\angle = 180° - 26° - 110° = 44°$

b) Find all the possible values for $\angle A$



amb.

$\frac{\sin 16°}{17} = \frac{\sin C°}{24}$

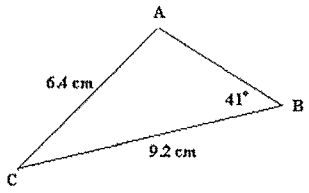
$\sin C° = 0.39$

$C \approx 23°$ or $157°$

$\therefore \angle A = 180° - 16° - 23° = 141°$

or $180° - 16° - 157° = 7°$

c) Find all the possible values for $\angle A$



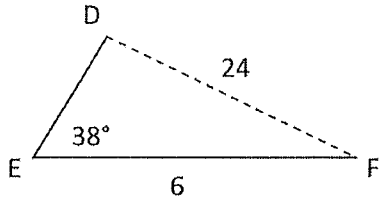
amb.

$\frac{\sin 41°}{6.4} = \frac{\sin A}{9.2}$

$\sin A = 0.943$

$A = 71°$ or $109°$

d) Find all the possible values for $\angle F$



not amb.

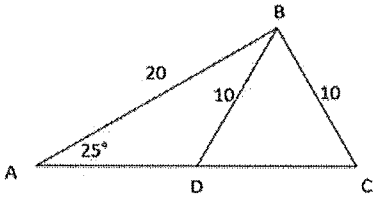
$\frac{\sin 38°}{24} = \frac{\sin D}{6}$

$\sin D = 0.154$

$D = 8.9°$

$\angle F = 180° - 8.9° - 38° = 133°$

e)Find all the possible values for $\angle C$



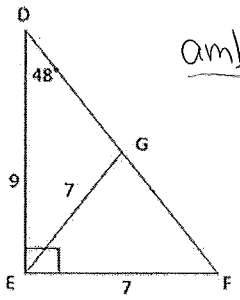
amb.

$\frac{\sin A}{10} = \frac{\sin C}{20}$

$\sin C = 0.845$

$C = 58° \text{ or } 122°$

f) Find all the possible values for $\angle DGE$



amb.

$\frac{\sin 48°}{7} = \frac{\sin \angle DGE}{9}$

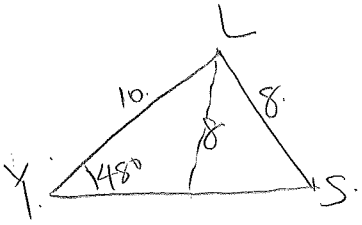
$\sin \angle G = 0.955$

$\angle G = 73° \text{ or } 107°$

2. A lighthouse at point L is 10 km from a yacht at point Y and 8 km from a sailboat at point B. From the yacht, the lighthouse and the sailboat are separated by an angle of 48º

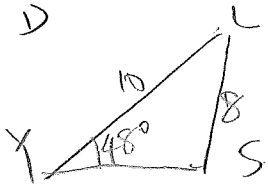
a) Is it necessary to ~~consider the ambi~~guous case? Explain.

Yes. $\overline{LS} < \overline{YL}$

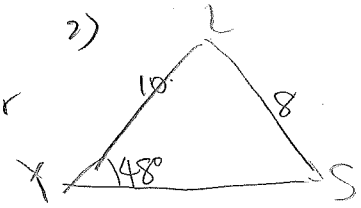


b) Sketch all possible diagrams for this situation.

1)



or 2)



c) Determine all possible the distances from the yacht to the sailboat, to the nearest tenth of a kilometre.

1) $\frac{\sin 48°}{8} = \frac{\sin \angle S}{10}$

$\sin \angle S = 0.929$

$\angle S = 68° \text{ or } 112°$

$\therefore \angle L = 64° \text{ or } 20°$

$\frac{\sin 64°}{YS} = \frac{\sin 48°}{8}$

$YS = 9.68$

$\frac{\sin 20°}{YS} = \frac{\sin 48°}{8}$

$YS = 3.68$

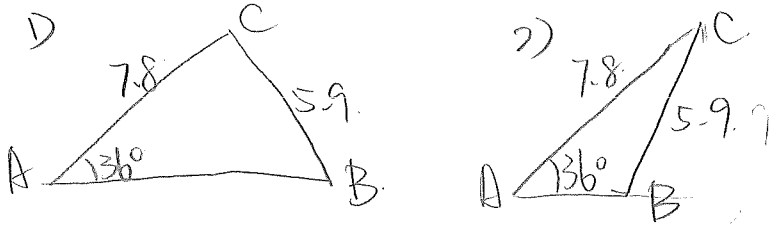
The distance is 9.7 or 3.7 km

3. Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of the cloud. Albert's rope is 7.8 m long and makes an angle of 36º with the ground. Belle's rope is 5.9 m long.

a) Is it necessary to consider the ambiguous case? Explain.

Yes $5.9\text{ m} < 7.8\text{m}$

b) Sketch all possible diagrams for this situation.



c) Determine all possible the distances between Albert and Belle to the nearest tenth of a meter.

$\frac{\sin B}{7.8} = \frac{\sin 36°}{5.9}$

$\sin B = 0.777$

$B = 51°$ or $129°$

$\therefore \angle C = 93°$ or $15°$

1) $\frac{\sin 93°}{AB} = \frac{\sin 36°}{5.9}$

$AB = 10.02$

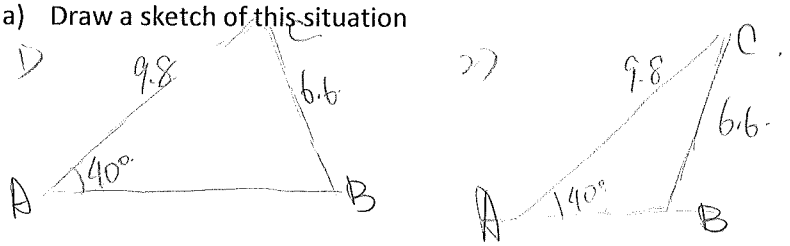
2) $\frac{\sin 15°}{AB} = \frac{\sin 36°}{5.9}$

$AB = 2.60$

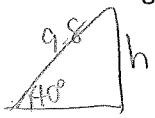
$\therefore AB =$ 10.0 or 2.6 m

5. A triangular plot of land is enclosed by a fence. Two sides of the fence are 9.8 m and 6.6 m long, respectively. The other side forms an angle of 40º with the 9.8 m side.

a) Draw a sketch of this situation



b) Calculate the height of the triangle to the nearest tenth. Compare it to the given sides.



$\sin 40° = \frac{h}{9.8}$

$h = 6.30\text{m}$

$6.3 < 9.8$

$6.3 < 6.6$

c) How many lengths are possible for the third side? Explain.

Two. include the ambiguous case since $6.6 < 9.8$

$AB =$ 9.5 or 5.6 m

d) Calculate all possible lengths to the nearest tenth of a meter.

$\frac{\sin 40°}{6.6} = \frac{\sin B}{9.8}$

$\sin B = 0.95$

$B = 73°$ or $107°$

$\therefore \angle C = 67°$ or $33°$

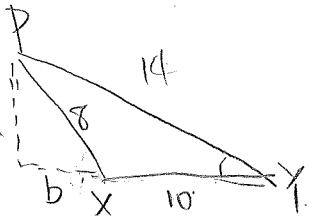
1) $\frac{\sin 67°}{AB} = \frac{\sin 40°}{6.6}$

$AB = 9.45$

2) $\frac{\sin 33°}{AB} = \frac{\sin 40°}{6.6}$

$AB = 5.59$

6. An eight metre telephone pole has a very bad lean and creates an angle greater than 90 degrees with the ground. A guide wire, 14 m long, is attached to the pole for support so the pole will not fall down. The guide wire is anchored in the ground at a point 10m from the base of the pole. Calculate the angle that the pole makes with the ground



Cosine law

$14^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos\theta$

$\cos\theta = -0.2$

$\theta = 101.5^\circ$

$\frac{\sin Y}{8} = \frac{\sin X}{14}$

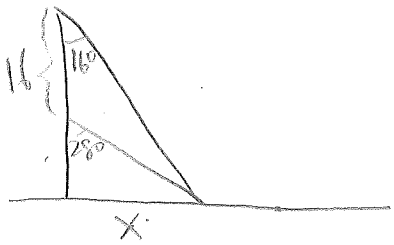
$\sin X = \frac{a}{8}$

$\sin Y = \frac{a}{14}$

$\frac{\frac{a}{14}}{8} = \frac{\frac{a}{8}}{14}$

$a = a$

7. A spider crawling down a wall spots its prey, a moth, on the ground at an angle of 16^0 with the wall. After crawling downward 16 cm, the moth still hasn't moved, but now the angle with the wall is 28^0. How far is the moth from the wall?

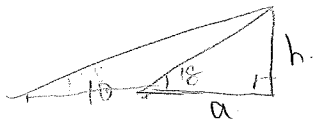


$\frac{x}{\tan 16^\circ} - \frac{x}{\tan 28^\circ} = 16$

$x\left(\frac{1}{\tan 16^\circ} - \frac{1}{\tan 28^\circ}\right) = 16$

$x = 9.96\text{ cm}$

8. While exploring the woods at the end of Bengal Road in Mira, two of Glace Bay's policemen, spotted a fire in the distance. From where they were standing, they estimated an angle of elevation of **15^0** to the top of the tower. Moving **10 m** closer to the tower, they now estimate the angle of elevation to be **18^0.** How high is the tower?



$\tan 18^\circ = \frac{h}{a}$ $\quad \tan 15^\circ = \frac{h}{10+a}$

$\tan 18^\circ \cdot a = h$ $\quad \tan 15^\circ (10+a) = h$

$\tan 18^\circ a = \tan 15^\circ (10+a)$

$(\tan 18^\circ - \tan 15^\circ) a = \tan 15^\circ \cdot 10$

$a = 47.03\text{m}$

$\tan 18^\circ = \frac{h}{a}$

$h = \tan 18^\circ \cdot a$

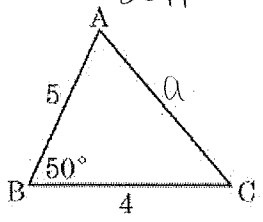
$h = 15.28\text{m}$

Name: Claire Li Date: 5.4

Math 9 Enriched: Section 6.5 Cosine Law

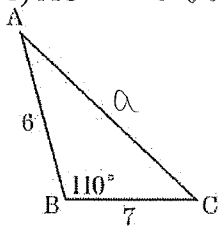
1. Given each triangle, find the value of the indicated side or angle.

a) AC = 3.91



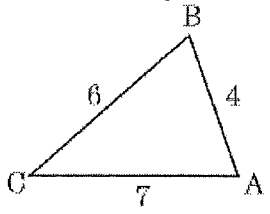
$a^2 = b^2 + c^2 - 2bc(\cos\angle 50°)$
$a^2 = 4^2 + 5^2 - 40(0.643)$
$a = 3.91$

b) AC = 10.66



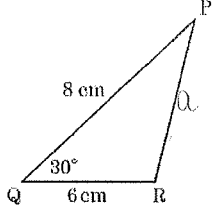
$a^2 = 6^2 + 7^2 - 84(\cos 110°)$
$a = 10.66$

c) $\angle B$ = 86°



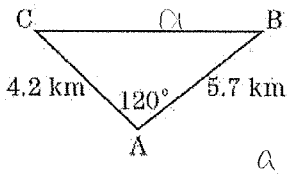
$b^2 = a^2 + c^2 - 2ac(\cos\angle B)$
$7^2 = 6^2 + 4^2 - 48(\cos\angle B)$
$1 = -16(\cos\angle B)$
$\angle B = 86°$

d) PR = 4.11



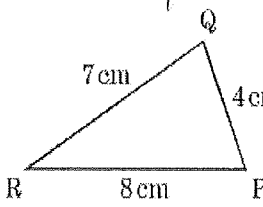
$a^2 = 8^2 + 6^2 - 2 \cdot 8 \cdot 6(\cos 30°)$
$a = 4.11$

e) $\angle B$ = 25°



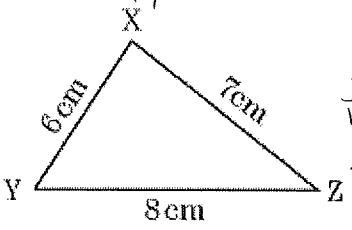
$a^2 = 5.7^2 + 4.2^2 - 2 \cdot 4.2 \cdot 5.7(\cos 120)$
$a^2 = 74.07$
$a = 8.61$
$\frac{a}{\sin 120°} = \frac{4.2}{\sin\angle B}$
$\sin\angle B = 0.422$
$\angle B = 25°$

f) $\angle Q$ = 89°



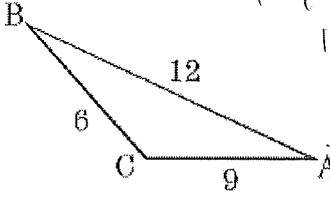
$8^2 = 4^2 + 7^2 - 8 \cdot 7(\cos\angle Q)$
$\frac{1}{56} = \cos\angle Q$
$\angle Q = 89$

g) $\angle z$ = 47°



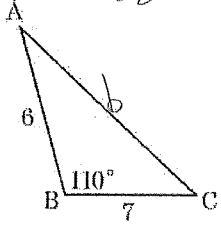
$b^2 = 7^2 + 8^2 - 112(\cos\angle z)$
$\frac{77}{112} = \cos\angle z$
$\angle z = 47°$

h) (Obtuse) $\angle C$ = 104°



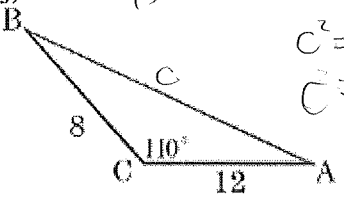
$12^2 = 6^2 + 9^2 - 108(\cos\angle C)$
$1 = -4(\cos\angle C)$
$\frac{1}{4} = \cos\angle C$
$\angle C = 104$

i) $\angle A$ = 38°



$b^2 = 6^2 + 7^2 - 84(\cos 110°)$
$b^2 = 113.73$
$b = 11.66$
$\frac{b}{\sin 110°} = \frac{7}{\sin\angle A}$
$\sin\angle A = 0.617$
$\angle A = 38°$

j) $\angle B$ = 43

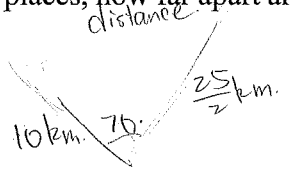


$c^2 = 8^2 + 12^2 - 192(\cos 110°)$
$c = 16.54$
$\frac{c}{\sin 110°} = \frac{12}{\sin\angle B}$
$\sin\angle B = 0.68$
$\angle B = 43°$

11:30 am − 9:00 am = 2h 30 min. = $2\frac{1}{2}$ h.

2. Two hikers start out from the same place at 9:00am. The first hiker walks at 4km/h and the second hiker walks at 5km/h. If the angle between the two hikers is 70° then, to 3 decimal places, how far apart are the hikers at 11:30am?

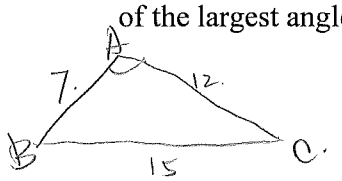
① $2\frac{1}{2} \times 4 = 10$ km. ② $2\frac{1}{2} \times 5 = \frac{25}{2}$ km.



$d^2 = \left(\frac{25}{2}\right)^2 + (10)^2 - 250(\cos 70°)$

$d = 13.067$ km

3. Triangle $\triangle ABC$ has sides of length 7, 12, and 15cm. To the nearest degree, what is the measure of the largest angle of the triangle?

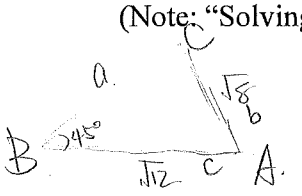


$15^2 = 7^2 + 12^2 - 168(\cos \angle A)$

$-0.19 = \cos \angle A$

$101° = \angle A$

4. Solve $\triangle ABC$ if $\angle B = 45°$, $c = \sqrt{12}$, and $b = \sqrt{8}$. Round your answers to 1 decimal place. (Note: "Solving" means finding all the missing angles and sides)



$\frac{\sqrt{8}}{\sin 45°} = \frac{\sqrt{12}}{\sin \angle C}$

$\sin \angle C = 0.866$

$\angle C = 60°$

$\therefore \angle A = 180° - 45° - 60° = 75°$

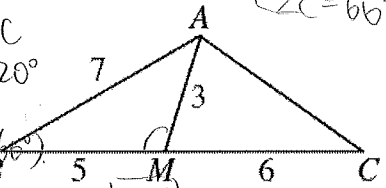
$a^2 = 8 + 2 + 4\sqrt{6} \cdot \cos 75°$

$a = 3.54$

$a = 3.5$
$b = \sqrt{8} = 2.8$
$c = \sqrt{12} = 3.5$

$\angle A = 75°$
$\angle B = 45°$
$\angle C = 60°$

5. In ΔABC, M is a point on BC such that $BM = 5$ and $MC = 6$. If $AM = 3$ and $AB = 7$, determine the exact value of AC.



$\angle AMC = 180° - 120° = 60°$

$7^2 = 5^2 + 3^2 - 30(\cos \angle AMB)$

$-0.5 = \cos \angle AMB$

$120° = \angle AMB$

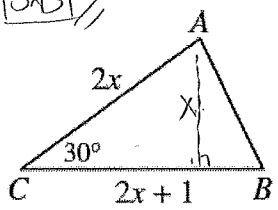
$AC^2 = 3^2 + 6^2 - 36(\cos 60°)$

$AC^2 = 9 + 36 - 36 \cdot \frac{1}{2}$

$AC^2 = 27$

$AC = 3\sqrt{3}$

6. In the diagram, $AC = 2x$, $BC = 2x + 1$ and $\angle ACB = 30°$. If the area of ΔABC is 18, what is the value of x?



$S_{\triangle ABC} = \frac{x(2x+1)}{2}$

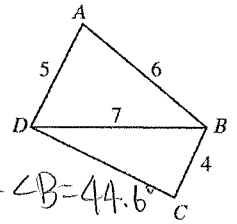
$36 = 3x^2 + x$

$3x^2 + x - 36 = 0$

$x = \frac{-1 \pm \sqrt{1 + 432}}{6}$

$x = 3.3$

7. In the diagram, $ABCD$ is a quadrilateral in which $\angle A + \angle C = 180°$. What is the length of CD?



$7^2 = 6^2 + 5^2 - 60(\cos \angle A)$

$\frac{1}{5} = \cos \angle A$

$\angle A = 78.463$

$\angle C = 180° - \angle A = 101.5°$

$\frac{4}{\sin \angle D} = \frac{7}{\sin 101.5}$

$\sin \angle D = 0.56$

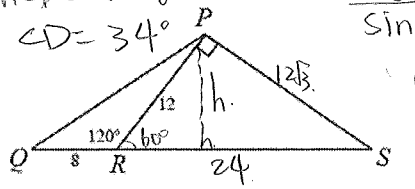
$\angle D = 34°$

$\therefore \angle B = 44.6°$

$\frac{CD}{\sin \angle B} = \frac{7}{\sin 101.5}$

$CD = 5$

8. What is the area of $\triangle QPS$?



$\angle PRS = 180° - 120° = 60°$

$PS = \sqrt{3} PR = 12\sqrt{3}$

$RS = 24$

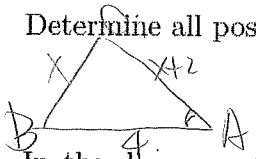
$S_{\triangle PSR} = \frac{h \cdot 24}{2}$

$\frac{12 \cdot 12\sqrt{3}}{2} = \frac{h \cdot 24}{2}$

$h = 6\sqrt{3}$

$S_{\triangle QPS} = \frac{6\sqrt{3}(8+24)}{2} = 96\sqrt{3}$

9. In $\triangle ABC$, $BC = 4$, $AB = x$, $AC = x + 2$, and $\cos(\angle BAC) = \dfrac{x+8}{2x+4}$. Determine all possible values of x.

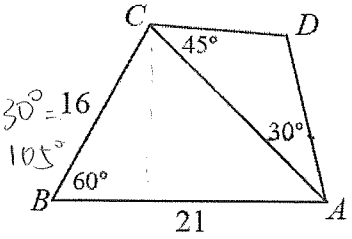


$x^2 = (x+2)^2 + 4^2 - 8(x+2)\left(\frac{x+8}{2x+4}\right)$

$x^2 = x^2 + 4x + 4 + 16 - 4x - 32$

$0 = -12$?

10. In the diagram, $AB = 21$ and $BC = 16$. Also, $\angle ABC = 60°$, $\angle CAD = 30°$, and $\angle ACD = 45°$. Determine the length of CD, to the nearest tenth.



$AC^2 = 16^2 + 21^2 - 672(\cos 60°)$

$AC^2 = 361$

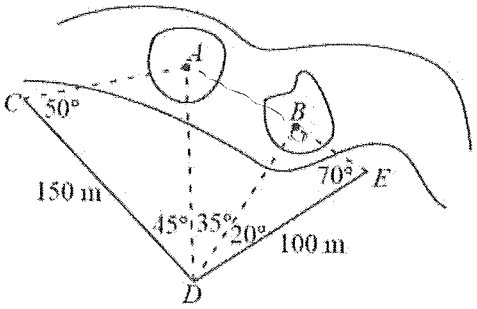
$AC = 19$

$\angle D = 180° - 45° - 30° = 16$

$\frac{CD}{\sin 30°} = \frac{19}{\sin 105°}$

$CD = 9.84$

11. In the diagram, points "A" and "B" are located on islands in a river full of rabid aquatic goats. Determine the distance from A to B, to the nearest meter.



$\angle CAD = 180° - 50° - 45° = 85°$

$\angle DBE = 180° - 20° - 70° = 90°$

$\frac{150}{\sin 85°} = \frac{AD}{\sin 50°}$

$AD = 115.3$

$\frac{100}{\sin 90°} = \frac{BD}{\sin 70}$

$BD = 93.97$

$AB^2 = (115.3)^2 + (93.97)^2 - 21669(\cos 35°)$

$AB = 66.1$ m

12. Find $\sin\theta$ if $\cos\theta = \dfrac{a}{a+2}$.

a) $\dfrac{2\sqrt{a+1}}{a+2}$ b) $\dfrac{4(a+1)}{a+2}$ c) $\dfrac{2\sqrt{a+1}}{a}$

d) $\dfrac{4(a+1)}{a}$ e) $\dfrac{\sqrt{a+1}}{2a}$

Solution 2:

$\sin^2\theta + \cos^2\theta = 1$

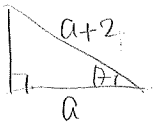
$x^2 + \frac{a^2}{(a+2)^2} = 1$

$1 - \frac{a^2}{(a+2)^2} = x^2$

$\frac{(a+2)^2 - a^2}{(a+2)^2} = x^2$

$\frac{2\sqrt{a+1}}{a+2} = x$

Solution 1:



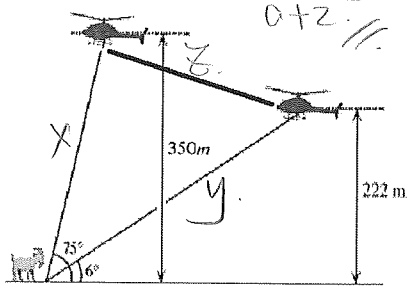
$\sqrt{(a+2)^2 - a^2}$

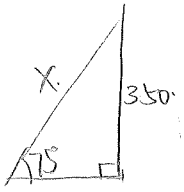
$= \sqrt{a^2 + 4a + 4 - a^2}$

$= 2\sqrt{a+1}$

$\therefore \sin\theta = \frac{2\sqrt{a+1}}{a+2}$

13. A helicoptop is flying at an altitude of 222m at constant speed. A stationary goat takes two measurements of the angle between the ground and helicopter. The first measurement the goat makes is 6° (222m) and the second measurement is 75° at an altitude of 350m. If the time between the two measurements is exactly 60seconds, then how fast is the helicopter flying in (km/hr)?





$\sin 75° = \frac{350}{x}$

$\frac{350}{\sin 75} = x$

$x = 362.35$m

$z^2 = x^2 + y^2 - 2xy(\cos 75°)$

$z = 2059.99$m

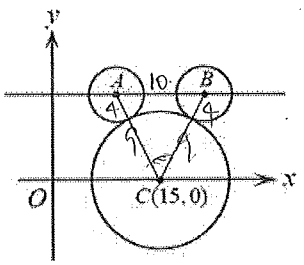
$\sin 6° = \frac{222}{y}$

$y = 2123.82$m

The helicopter is flying in

123.60 km/hr

14. In the diagram, the large circle has radius 9 and centre C(15,0). The small circles have radius 4 and centres at "A" and "B". Two little bugs travelling at the same speed goes from point "A" to point "B". The first bug takes 10seconds. The second bug travels from "A" to "C" and then back to "B" taking 26seconds. What are the coordinates of points "A" and "B"? What is $\angle ACB$?



$4+9+9+4=26$

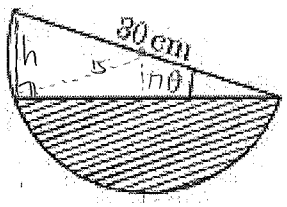
$16^2 = 13^2 + 13^2 - 338(\cos\angle ACB)$

$\frac{238}{338} = \cos\angle ACB$

$\angle ACB = 45^\circ$

15. Water in a hemispherical bowl with a diameter of 30 cm begins to pour out when the bowl is tilted to an angle θ equal to 17°. How deep is the water in the bowl? Round off your answer to the nearest tenth of a centimetre.

a) 4.4 b) 5.7 c) 10.4
d) 10.6 e) 25.6

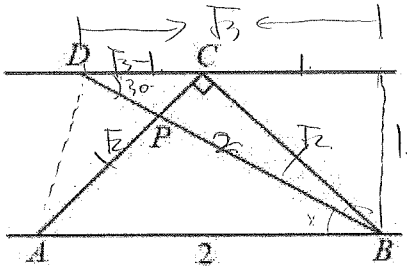


$\sin\theta = \frac{h}{30}$

$h = 8.77$

$15 - h\frac{1}{2} = 10.6$ cm

16. Points A, B, C, and D are arranged, as shown, with AB parallel to DC and P the point of intersection of AC and BD. Also, $\angle ACB = 90^\circ$, $AC = CB$, $AB = BD = 2$. Determine the measure of $\angle DBC$.



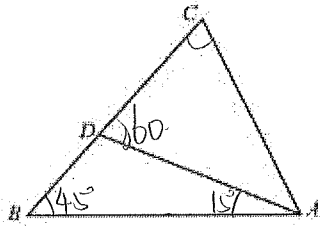
$AC = CB = \frac{2}{\sqrt{2}} = \sqrt{2}$.

$(\sqrt{3}-1)^2 = 2^2 + 2 - 4\sqrt{2}\cos\angle DBC$

$\angle DBC = 15^\circ$

17. In triangle ABC, $\angle ABC = 45^\circ$. Point D is on $\overline{BC}$ so that $2 \cdot BD = CD$ and $\angle DAB = 15^\circ$. Find $\angle ACB$. $= X$

(A) 54° **(B)** 60° **(C)** 72° **(D)** 75° **(E)** 90°



$\frac{AD}{\sin 45^\circ} = \frac{BD}{\sin 15^\circ}$

$\frac{\sin 15^\circ AD}{\sin 45} = BD$

$\frac{CD}{\sin\angle DBC} = \frac{AD}{\sin\angle ACB}$

$\frac{2\sin 15^\circ}{\sin 45^\circ} = \frac{\sin\angle DBC}{\sin\angle ACB}$

$\sin\angle ACB = \sin(120 - X)$

$X = 15^\circ$

18. Challenge:

What is $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 358^\circ + \cos 359^\circ$?

$\cos X^\circ + \cos(180 - X)^\circ = 0$

$\cos 1^\circ + \cos 179^\circ = 0$

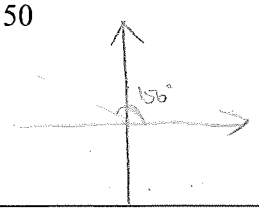
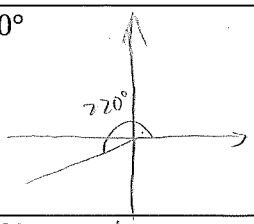
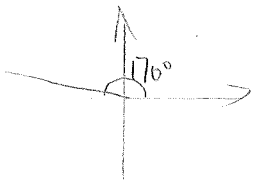
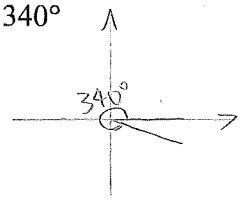
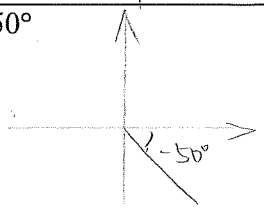
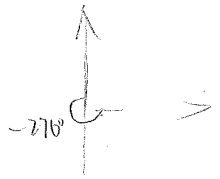
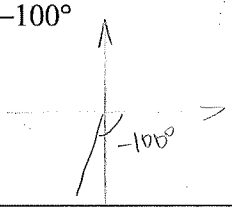
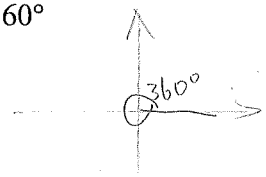
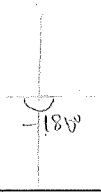
$\cos 181^\circ + \cos 359^\circ = 0$

$\therefore \cos 1^\circ + \cos 2^\circ + \cdots + \cos 359^\circ = 0$

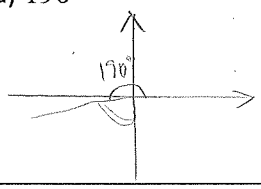
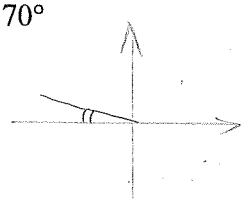
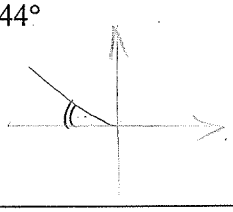
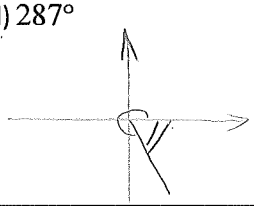
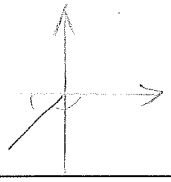
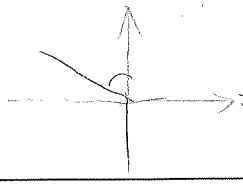
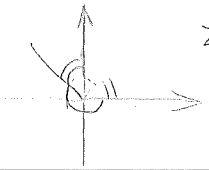
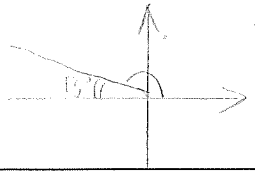
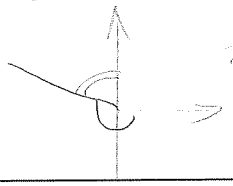
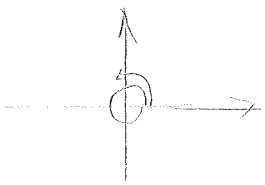
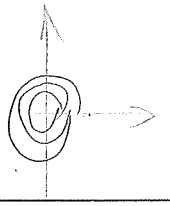
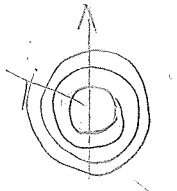
Name: Claire Li Date: 5.7

Math 9 Enriched: Assignment 6.6 Angles in Standard Position

1. Draw each of the angles in standard position

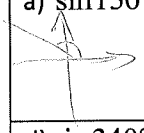
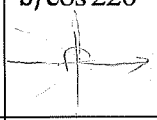
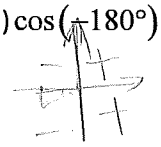
a) 150 	b) 220° 	c) 170° 
d) 340° 	e) −50° 	f) −270° 
g) −100° 	h) 360° 	i) −180° 

2. Find the reference angle for each of the following angles in standard position:

a) 190°  270°−190° = 80°	b) 170°  180°−170° = 10°	c) 144°  180°−144° = 36°
d) 287°  360°−287° = 73°	e) −140°  −180−(−140°) = −40°	f) −199°  270−199 = 71°
g) −217°  270°−217° = 53°	h) 167°  180−167 = 13°	i) −213°  270°−213° = 57°
j) 455°  85°	k) 1080°  0°	L) −2000°  1500 90°−20° = 70°

A S T C

3. Indicate whether if each of the following functions will be either positive or negative

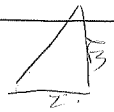
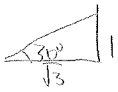
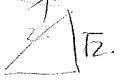
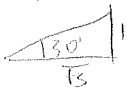
a) $\sin 150°$  Quadrant 2 positive //	b) $\cos 220°$  Quadrant 3. negative //	c) $\tan 170°$ Quadrant 2. negative //
d) $\sin 340°$ Quadrant 4 negative //	e) $\cos(-50°)$ Quadrant 4 positive //	f) $\sin(-270°)$ Quadrant 1. positive //
g) $\tan(-100°)$ Quadrant 3 positive //	h) $\sin 360°$ 0 //	i) $\cos(-180°)$  negative //
j) $\sin(170°)$ Quadrant 2 positive //	k) $\cos(233°)$ Quadrant 3. negative //	L) $\tan(155°)$ Quadrant 2 negative //
m) $\cos(230°)$ Quadrant 3 negative //	n) $\tan(680°)$  Quadrant 4 negative //	O) $\tan(290°)$ Quadrant 4. negative //

A S T C

4. Given the ratios, which two quadrants will θ be in ?

a) $\sin\theta = -\frac{1}{2}$ Quadrant 3 & 4.	b) $\cos\theta = \frac{\sqrt{3}}{2}$ Quadrant 2 & 3	c) $\tan\theta = \frac{-\sqrt{3}}{2}$ Quadrant 2 & 4
d) $\sin\theta = \frac{\sqrt{2}}{2}$ Quadrant 3 & 4.	e) $\cos\theta = -\frac{\sqrt{2}}{2}$ Quadrant 2 & 3.	f) $\tan\theta = -\sqrt{3}$ Quadrant 2 & 4.
g) $\tan\theta = -1$ Quadrant 2 & 4	h) $\sin\theta = -\frac{\sqrt{2}}{2}$ Quadrant 3 & 4.	i) $\tan\theta = -\frac{1}{\sqrt{3}}$ Quadrant 2 & 4.

5. Solve for θ between $0 \le \theta \le 360°$

a) $\sin\theta = -\frac{1}{2}$  $\theta = 30°$ Quadrant 3 or 4 $180° + 30° = 210°$ $360° - 30° = 330°$ 210° or 330°	b) $\cos\theta = \frac{\sqrt{3}}{2}$  $\theta = 30°$ Quadrant 1 or 3. 30° $180° + 30° = 210°$ 30° or 210°	c) $\tan\theta = \frac{-\sqrt{3}}{2}$  $\theta = 41°$ Quadrant 2 or 4. 139° or 319°
d) $\sin\theta = \frac{\sqrt{2}}{2}$  $\theta = 45°$ Quadrant 1 or 2. 45° or 135°	e) $\cos\theta = -\frac{\sqrt{2}}{2}$  $\theta = 45°$ Quadrant 2 or 4. 135° or 315°	f) $\tan\theta = -\sqrt{3}$  $\theta = 60°$ Quadrant 2 or 3. 120° or 240°
g) $\tan\theta = -1$  $\theta = 45°$ Quandrant 2 or 3. 135° or 225°	h) $\sin\theta = -\frac{\sqrt{2}}{2}$  $\theta = 45°$ Quandrant 3 or 4 225° or 315°	i) $\tan\theta = -\frac{1}{\sqrt{3}}$  $\theta = 30°$ Quadrand 2 or 3. 150° or 210°

6. If $\sin\theta > 0$ and $\cos\theta > 0$, then which quadrant is θ in?

S | A
T | C

Quadrant 1.

7. If $\sin\theta < 0$ and $\cos\theta > 0$, then which quadrant is θ in?

Quadrant 3.

8. If $\tan\theta > 0$ and $\cos\theta < 0$, then which quadrant is θ in?

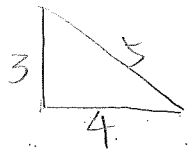
Quadrant 4

9. If $\tan\theta < 0$ and $\cos\theta < 0$, then which quadrant is θ in?

Quadrant 2.

10. If $\sin\theta = \frac{3}{5}$, then what is the value of $\cos\theta$?

$\cos\theta = \frac{a}{5}$



11. If $\cos\theta = \frac{\sqrt{3}}{7}$, then what is the value of $\sin\theta$?

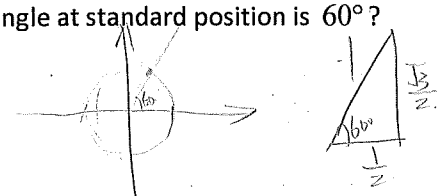
$\sin\theta = \frac{2}{7}$

(?) How to determine whether $\sin\theta$ is positive or negative

12. If $\tan\theta = \frac{-\sqrt{2}}{3}$, then what is the value of $\sin\theta$?

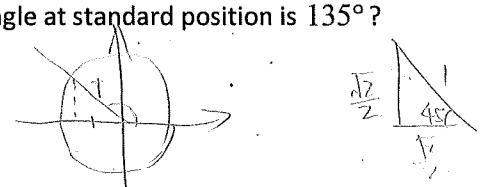
$\sin\theta = \frac{\sqrt{2}}{\sqrt{11}}$

13. If point "P" is on an unit circle and at the end of the terminal arm, then what are the coordinates of 'P" if the angle at standard position is $60°$?



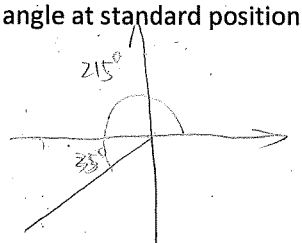
$P(\frac{1}{2}, \frac{\sqrt{3}}{2})$

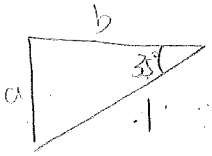
14. If point "P" is on an unit circle and at the end of the terminal arm, then what are the coordinates of 'P" if the angle at standard position is $135°$?



$P(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

15. If point "P" is on an unit circle and at the end of the terminal arm, then what are the coordinates of 'P" if the angle at standard position is $215°$?





$\sin 35° = \frac{a}{1}$

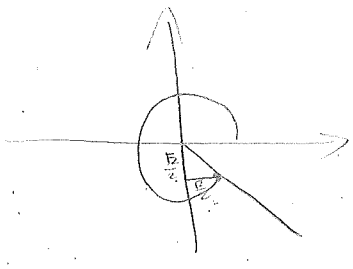
$a = 0.57$

$\cos 35° = \frac{b}{1}$

$b = 0.82$

$P(-0.82, -0.57)$

16. If point "P" is on an unit circle and at the end of the terminal arm, then what are the coordinates of 'P" if the angle at standard positiofsn is $315°$?



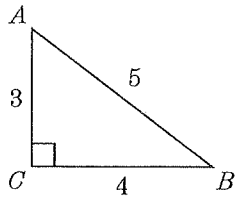
$P(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

Math 9 Enriched

Trigonometry Review

Name Claire Li Date May 14th

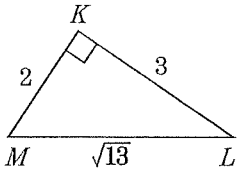
1. Find $\sin \angle A$.



$\sin \angle A = \frac{4}{5}$

$= 0.8$

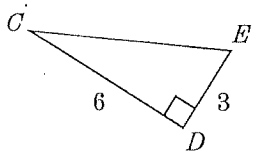
2. Find $\sin \angle M$.



$\sin \angle M = \frac{3}{\sqrt{13}}$

$= 0.83$

3. Find $\sin \angle E$.

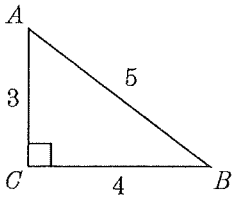


$CE = 3\sqrt{5}$

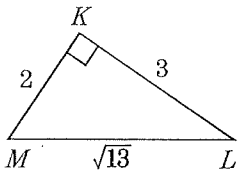
$\sin \angle E = \frac{6}{3\sqrt{5}}$

$= 0.89$

4. $\cos \angle A = \frac{3}{5}$

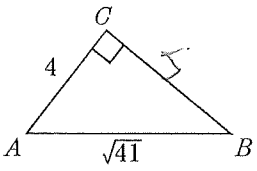


5. $\sin \angle M = \frac{3\sqrt{13}}{13}$



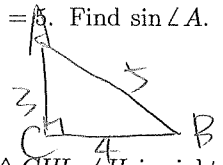
$\frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

6. $\sin \angle A = \frac{5\sqrt{41}}{41}$



$\frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

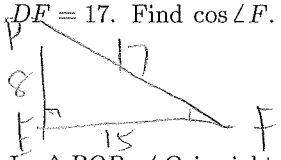
7. In $\triangle ABC$, $m\angle C = 90°$, $AC = 3$, $BC = 4$, and $AB = 5$. Find $\sin \angle A$.



$\sin \angle A = \frac{4}{5}$

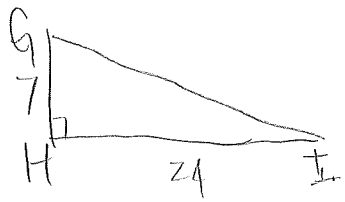
$= 0.8$

8. In $\triangle DEF$, $\angle E$ is right, $DE = 8$, $FE = 15$, and $DF = 17$. Find $\cos \angle F$.



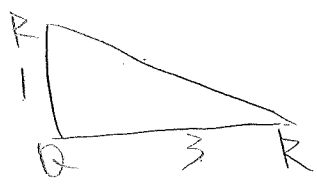
$\cos \angle F = \frac{15}{17}$

9. In $\triangle GHI$, $\angle H$ is right, $GH = 7$, and $IH = 24$. Find $\tan \angle I$.



$\tan \angle I = \frac{7}{24}$

10. In $\triangle PQR$, $\angle Q$ is right, $PQ = 1$, and $QR = 3$. Find $\tan \angle R$.



$\tan \angle R = \frac{1}{3}$

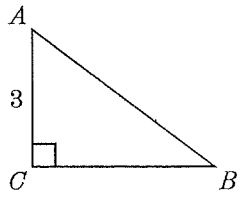
11. If $\cos \angle P = 0.3228$, find $m\angle P$ to the nearest 10 minutes.

$m\angle P = 71.1°$

12. If $\cos \angle V = 0.3987$, find $m\angle V$ to the nearest tenth of a degree.

$m\angle V = 66.5°$

13. If $\sin \angle B = \frac{2}{3}$, find AB.

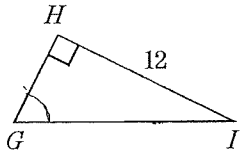


$\sin \angle B = \frac{2}{3}$

$\frac{3}{AB} = \frac{2}{3}$

$AB = \frac{9}{2}$

14. If $\sin \angle G = \frac{12}{13}$, find GI.

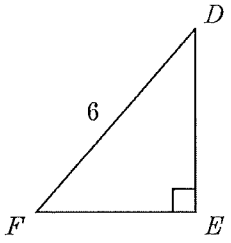


$\sin \angle G = \frac{12}{13}$

$\frac{12}{GI} = \frac{12}{13}$

$GI = 13$

15. If $\sin \angle F = \frac{5}{6}$, find FE.



$\sin \angle F = \frac{5}{6}$

$\frac{DE}{6} = \frac{5}{6}$

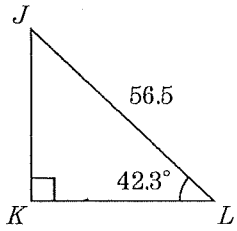
$DE = 5$

$FE = \sqrt{6^2 - 5^2} = \sqrt{11}$

16. If $\cos \angle F = \frac{1}{6}$, find DE.

$DE = \sqrt{35}$

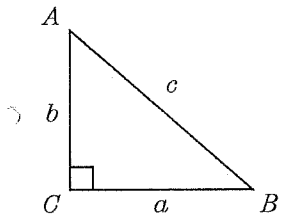
17. Find JK to the nearest tenth.



$\sin 42.3 = \frac{JK}{56.5}$

$JK = 38.0$

18. If $a = 2$ and $b = 5$, find $m\angle A$ to the nearest ten minutes.

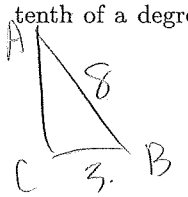


$c = \sqrt{5^2 + 2^2} = \sqrt{29}$

$\sin \angle A = \frac{2}{\sqrt{29}}$

$\angle A = 21.8°$

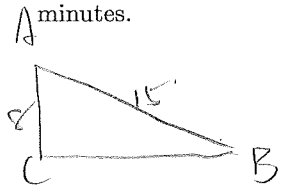
19. If $a = 3$ and $c = 8$, find $m\angle B$ to the nearest tenth of a degree.



$\cos \angle B = \frac{3}{8}$

$\angle B = 68.0°$

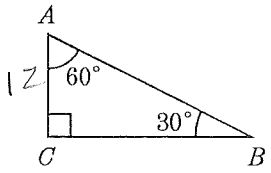
20. If $b = 8$ and $c = 15$, find $m\angle B$ to the nearest ten minutes.



$\sin \angle B = \frac{8}{15}$

$\angle B = 32.2°$

21. If $AC = 12$, find BC.



$BC = AC \cdot \sqrt{3}$

$= 12\sqrt{3}$ //

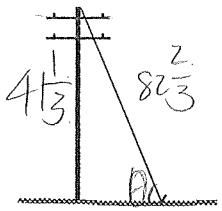
22. If $BC = 9$, find AB.

$AB = \frac{2BC}{\sqrt{3}} = \frac{2 \cdot 9\sqrt{3}}{3} = 6\sqrt{3}$ //

23. If $AB = 6$, find AC.

$AC = \frac{AB}{2} \cdot \sqrt{3}$ //

24. A 82 foot 8 inch long wire is attached to the top of a telephone pole 41 feet 4 inches tall. What is the exact measure of the angle the wire makes with the ground?

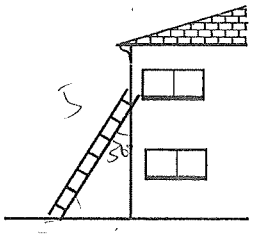


$\sin\theta = \frac{41\frac{1}{3}}{82\frac{2}{3}}$

$= \frac{1:1:1}{2}$

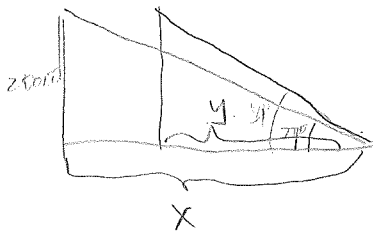
$\theta = 30°$

25. A 5 foot long ladder leans against a house forming a 30° angle with the house. Exactly how far is the base of the ladder from the house?



$\frac{5}{2}$ foot.

26. An airplane is flying at a constant altitude of 20,000 feet directly towards an observer stationed on the ground. At 2:00 the observed angle of elevation of the plane is 27°. One minute later, the angle of elevation is 51°. What is the plane's approximate ground speed in miles per hour?



$\tan 27° = \frac{20000}{x}$

$\frac{20000}{\tan 27°} = x$

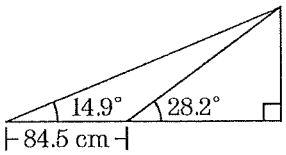
$\tan 51° = \frac{20000}{y}$

$x - y = \frac{20000}{\tan 27°} - \frac{20000}{\tan 51°} \quad \frac{20000}{\tan 51°} = y$

$= 23056$ feet/minute

$23056 \cdot \frac{60}{5280}$ 262 m/h //

27. Find the value of x as indicated in the diagram.



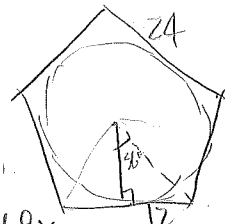
$\frac{84.5+X}{\tan 28.2^\circ} = \frac{X}{\tan 14.9^\circ}$

$\frac{42.31+X}{\tan 28.2^\circ} = \frac{X}{\tan 14.9^\circ}$

$\tan 28.2X = 12.056 + \tan 14.9X$

$X = 44.63$

28. The side of a regular pentagon is 24 centimeters. Approximate the radius of the circumscribed circle.



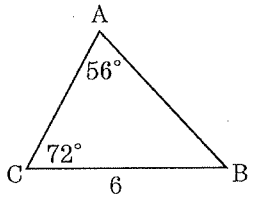
$\tan 36^\circ = \frac{12}{r}$

$\frac{12}{\tan 36^\circ} = r$

16.5 cm

29. A ship leaves its home port and sails on a bearing of N 29° 20′ E. A second ship leaves the same port at the same time and sails on a bearing of S 60° 40′ E. If the first ship sails at 32.0 miles per hour and the second ship sails at 29.0 miles per hour, find the approximate distance between the two ships after $3\frac{1}{2}$ hours.

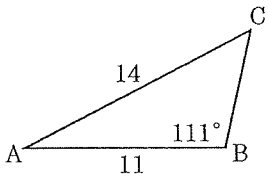
30. Calculate the length of AB in △CAB to 1 decimal place.



$\frac{\sin 56^\circ}{6} = \frac{\sin 72^\circ}{AB}$

$AB = 6.9$

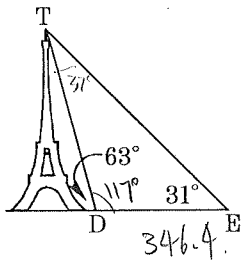
31. Calculate the measure of ∠C in △BCA to the nearest tenth of a degree.



$\frac{\sin \angle C}{11} = \frac{\sin 111^\circ}{14}$

$\angle C = 47.2^\circ$

32. Daniel is at point D and Edward is at point E. Daniel looks up at an angle of 63° from DE to see the top of the Eiffel Tower at T, and Edward looks up at it with an angle of 31°. The length of DE is 346.4 m. How high is the Eiffel Tower to the nearest tenth of a metre?



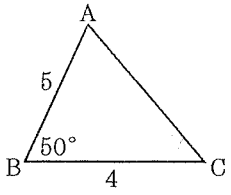
$\frac{\sin 32^\circ}{346.4} = \frac{\sin 31^\circ}{TD}$

$TD = 336.67$ m.

$\sin \angle 63^\circ = \frac{h}{TD}$

299.98 m $= h$

33. Calculate the length of AC in $\triangle BAC$ to 1 decimal place.

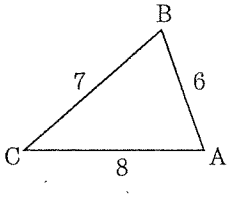


$AC^2 = 5^2 + 4^2 - 40 \cdot \cos 50°$

$AC^2 = 25 + 16 - 25.71$

$AC = \boxed{3.91}$

34. Calculate the measure of $\angle A$ in $\triangle CBA$ to the nearest tenth of a degree.

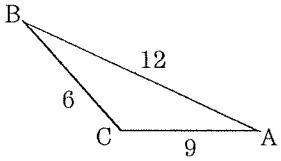


$7^2 = 6^2 + 8^2 - 96 \cdot \cos \angle A$

$\frac{51}{96} = \cos \angle A$

$\angle A = \boxed{57.9°}$

35. Calculate the measure of $\angle A$ in $\triangle CBA$ to the nearest tenth of a degree.



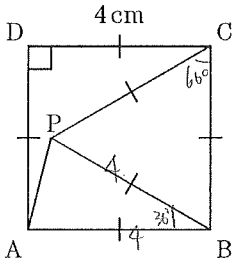
$6^2 = 12^2 + 9^2 - 24 \cdot 9 \cos \angle A$

$13 = 24 \cos \angle A$

$\frac{13}{24} = \cos \angle A$

$\angle A = \boxed{57.2°}$

36. ABCD is a square with side length 4 cm and $\triangle PBC$ is equilateral as shown. Determine the length of AP to the nearest tenth of a centimetre.



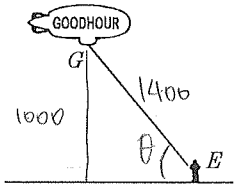
$AP^2 = 4^2 + 4^2 - 32 \cos 30°$

$AP^2 = 32 - 32 \cdot \frac{\sqrt{3}}{2}$

$AP^2 = 32 - 16\sqrt{3}$

$AP = \boxed{2.07 \text{ cm}}$

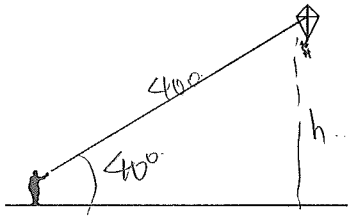
37. The Goodhour Blimp was flying overhead the other day at a height of 1000 meters. Egbert, standing at point E, was 1400 meters from someone riding in the blimp, at point G. What was the angle of elevation observed by Egbert? Answer to the nearest tenth of a degree.



$\sin \angle \theta = \frac{1000}{1400}$

$\angle \theta = \boxed{45.6°}$

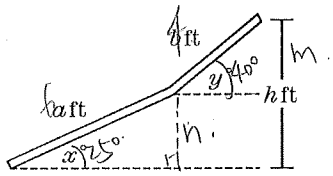
38. When I went kite flying the other day, I managed to let out an entire roll of string (400 feet). If the string, when pulled tight, formed a 40° angle with the ground, about how high was the kite?



$\sin 40° = \frac{h}{400}$

$\boxed{257.12} \text{ h.}$

39. A water slide is made up of two parts with different slopes as shown. The top part is 4 ft long ($b = 4$) and sloped at a 40° angle. The bottom part is 6 ft long ($a = 6$) and sloped at a 25° angle. How high is the top of the slide (h)?



$\sin 40° = \frac{m}{4}$

$m = 4 \cdot \sin 40°$

$\sin 25° = \frac{n}{6}$

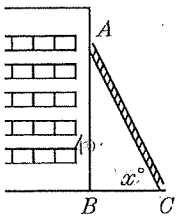
$n = 6 \cdot \sin 25°$

$h = m + n$

$= 4 \cdot \sin 40° + 6 \cdot \sin 25°$

$= 5.11$ ft

40. A ladder reaches a window 40 ft (AB = 40) above the ground and makes a 55° angle ($\angle ACB = 55$) with the ground.



a) $\sin 55° = \frac{40}{AC}$

$\frac{40}{\sin 55°} = AC$

$AC = 48.8$ ft

a) What is the length of the ladder to the nearest tenth of a foot?

b) If the ladder were moved so that it reaches 46 ft above the ground, what is the new angle with the ground to the nearest tenth of a degree?

c) To the nearest tenth of a foot, how far must the bottom of the ladder be moved in to form the new angle?

b) $\sin \angle X = \frac{46}{48.8}$

$\angle X = 70.4°$

c) $BC_{original} = \cos 55° \cdot AC$

$= 28.01$

$BC_{new} = \cos 70.4° \cdot AC$

$= 16.38$

$28.01 - 16.38 =$ 11.6 ft

Math 9 Enriched Trigonometry Review Mr. Young 19/03/2010

Answer List

1. $\frac{4}{5}$	2. $\frac{3\sqrt{13}}{13}$	3. $\frac{2\sqrt{5}}{5}$
4. A	5. M	6. A
7. $\frac{4}{5}$	8. $\frac{15}{17}$	9. $\frac{7}{24}$
10. $\frac{1}{3}$	11. $71^\circ 10'$	12. 66.5°
13. $\frac{9}{2} = 4.5$	14. 13	15. $\sqrt{11}$
16. $\sqrt{35}$	17. 38.0	18. $21^\circ 50'$
19. 68.0°	20. $32^\circ 10'$	21. $12\sqrt{3}$
22. $6\sqrt{3}$	23. 3	24. 30°
25. 2.5 ft	26. 262 mph	27. 44.6 cm
28. 20.4 cm	29. 151 mi	30. 6.9
31. 47.2°	32. 300.0 m	33. 3.9
34. 57.9°	35. 29.0°	36. 2.1 cm
37.	38.	39.
40.		

Catalog List

1. TRI MA 1	2. TRI MA 37	3. TRI MA 61
4. TRI MD 2	5. TRI MD 25	6. TRI MD 49
7. TRI MB 1	8. TRI MB 17	9. TRI MB 30
10. TRI MB 54	11. TRI ME 30	12. TRI ME 42
13. TRI MF 5	14. TRI MF 25	15. TRI MF 41
16. TRI MF 48	17. TRI MG 10	18. TRI MG 34
19. TRI MG 46	20. TRI MG 54	21. TRI MI 5
22. TRI MI 20	23. TRI MI 35	24. TRI ML 6
25. TRI ML 33	26. TRI ML 45	27. TRI ML 59
28. TRI ML 75	29. TRI ML 84	30. AW1 HG 1
31. AW1 HG 8	32. AW1 HG 11	33. AW1 HH 1
34. AW1 HH 6	35. AW1 HH 8	36. AW1 HH 21
37.	38.	39.
40.		